UNIT – I - Force Analysis

(1) Introduction: If the acceleration of moving links in a mechanism is running with considerable amount of linear and/or angular accelerations, inertia forces are generated and these inertia forces also must be overcome by the driving motor as an addition to the forces exerted by the external load or work the mechanism does.

(2) Newton's Law: First Law Everybody will persist in its state of rest or of uniform motion (constant velocity) in a straight line unless it is compelled to change that state by forces impressed on it. This means that in the absence of a non-zero net force, the center of mass of a body either is at rest or moves at a constant velocity. **Second Law** A body of mass *m* subject to a force **F** undergoes an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force and inversely proportional to the mass, i.e., $\mathbf{F} = m\mathbf{a}$. Alternatively, the total force applied on a body is equal to the time derivative of linear momentum of the body. **Third Law** The mutual forces of action and reaction between two bodies are equal, opposite and collinear. This means that whenever a first body exerts a force **F** on a second body, the second body exerts a force −**F** on the first body. **F** and −**F** are equal in magnitude and opposite in direction. This law is sometimes referred to as the *action-reaction law*, with **F** called the "action" and −**F** the "reaction"

(3) Types of force Analysis:

- Equilibrium of members with two forces
- Equilibrium of members with three forces
- Equilibrium of members with two forces and torque
- Equilibrium of members with two couples.
- Equilibrium of members with four forces.

(4) Principle of Super Position: Sometimes the number of external forces and inertial forces acting on a mechanism are too much for graphical solution. In this case we apply the method of superposition. Using superposition the entire system is broken up into (n) problems, where n is the number of forces, by considering the external and inertial forces of each link individually. Response of a linear system to several forces acting simultaneously is equal to the sum of responses of the system to the forces individually. This approach is useful because it can be performed by graphically.

(5) Free Body Diagram: A free body diagram is a pictorial representation often used by physicists and engineers to analyze the forces acting on a body of interest. A free body diagram shows all forces of all types acting on this body. Drawing such a diagram can aid in solving for the unknown forces or the equations of motion of the body. Creating a free body diagram can make it easier to understand the forces, and torques or moments, in relation to one another and suggest the proper concepts to apply in order to find the solution to a problem. The diagrams are also used as a conceptual device to help identify the internal forces—for example, shear forces and bending moments in beams—which are developed within structures.

(6) D'Alemberts Principle: D'Alembert's principle, also known as the **Lagrange–d'Alembert principle**, is a statement of the fundamental classical laws of motion. It is named after its discoverer, the French physicist and mathematician Jean le Rond d'Alembert. The principle states that the sum of the differences between the forces acting on a system and the time derivatives of the momenta of the system itself along any virtual displacement consistent with the constraints of the system is zero.

(7) Dynamic Analysis of Four bar Mechanism: A **four-bar linkage** or simply a **4-bar** or **four-bar** is the simplest movable linkage. It consists of four rigid bodies (called bars or links), each attached to two others by single joints or pivots to form closed loop. Four-bars are simple mechanisms common in mechanical engineering machine design and fall under the study of kinematics.

- □ Dynamic Analysis of Reciprocating engines.
- □ Inertia force and torque analysis by neglecting weight of connecting rod.
- □ Velocity and acceleration of piston.
- □ Angular velocity and Angular acceleration of connecting rod.
- □ Force and Torque Analysis in reciprocating engine neglecting the weight of connecting rod.
- □ Equivalent Dynamical System
- □ Determination of two masses of equivalent dynamical system

(8) Turning Moment Diagram: The turning moment diagram is graphical representation of the turning moment or crank effort for various positions of crank.

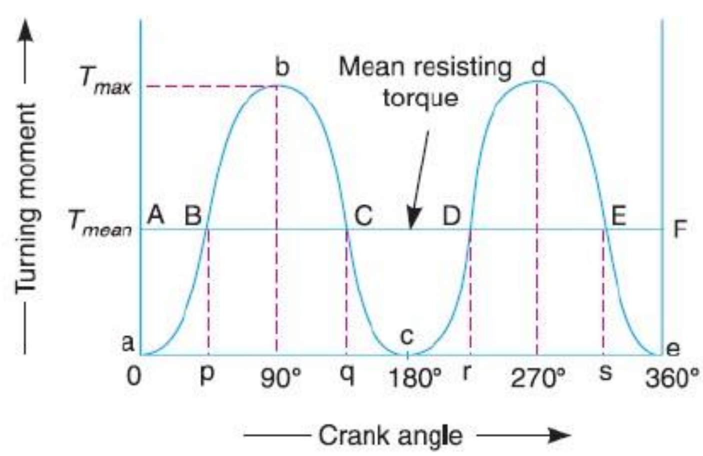
<u>LECTURE-2</u>

(9) Single cylinder double acting engine:

A turning moment diagram for a single cylinder double acting steam engine is shown in Fig. The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle.

the turning moment on the crankshaft,

$$T = F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$



Turning moment diagram for a single cylinder, double acting steam engine.

where F_P = Piston effort,

r = Radius of crank,

n = Ratio of the connecting rod length and radius of crank, and

θ = Angle turned by the crank from inner dead centre.

From the above expression, we see that the turning moment (T) is zero, when the crank angle (θ) is zero. It is maximum when the crank angle is 90° and it is again zero when crank angle is 180°.

This is shown by the curve *abc* in Fig. and it represents the turning moment diagram for outstroke. The curve *cde* is the turning moment diagram for instroke and is somewhat similar to the curve *abc*.

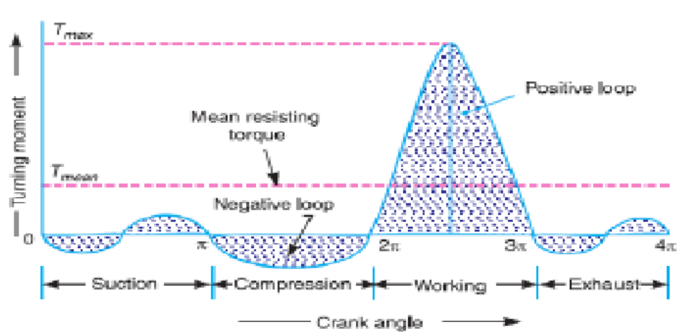
Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution. In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line *AF*. The height of the ordinate *aA* represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle *aAFe* is proportional to the work done against the mean resisting torque.

For flywheel, have a look at your tailor's manual sewing machine.

LECTURE-3

(10) Turning moment diagram for 4-stroke I.C engine:

A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, *i.e.* 720° (or 4 π radians).

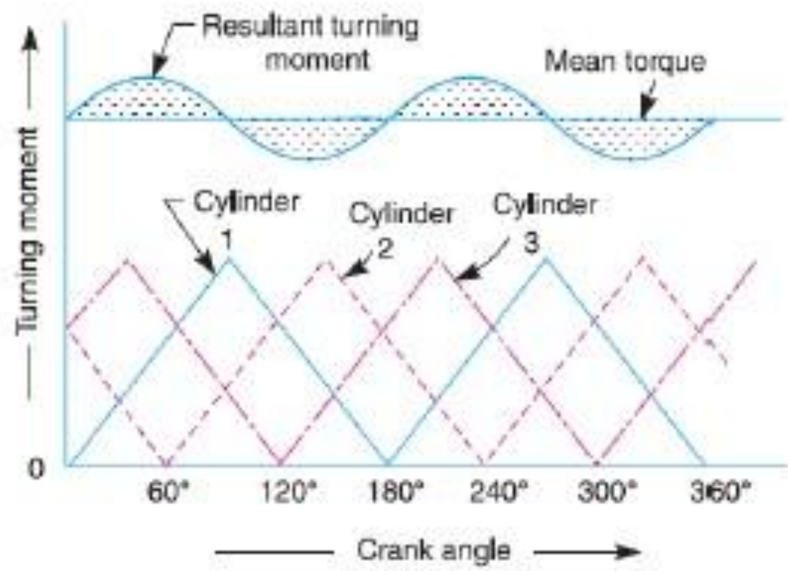


Turning moment diagram for a four stroke cycle internal combustion engine.

Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in Fig. 16.2. During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained. During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases, therefore a negative loop is formed. It may be noted that the effect of the inertia forces on the piston is taken into account in Fig.

(11) Turning moment diagram for a multi cylinder engine:

A separate turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in Fig. The resultant turning moment diagram is the sum of the turning moment diagrams for the three cylinders. It may be noted that the first cylinder is the high pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder. The cranks, in case of three cylinders, are usually placed at 120° to each other.

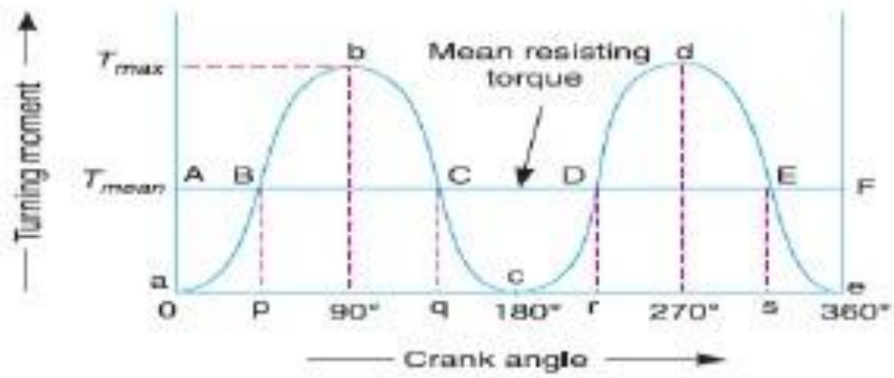


Turning moment diagram for a multi-cylinder engine.

Lecture-4

(12) Fluctuation of Energy:

The difference in the kinetic energies at the point is called the maximum fluctuation of energy.



The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in Fig. We see that the mean resisting torque line *AF* cuts the turning moment diagram at points *B*, *C*, *D* and *E*. When the crank moves from *a* to *p*, the work done by the engine is equal to the area *aBp*, whereas the energy required is represented by the area *aABp*. In other words, the engine has done less work (equal to the area *a AB*) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases. Now the crank moves from *p* to *q*, the work done by the engine is equal to the area *pBbCq*, whereas the requirement of energy is represented by the area *pBCq*. Therefore, the engine has done more work than the requirement. This excess work (equal to the area *BbC*) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from *p* to *q*.

Similarly, when the crank moves from *q* to *r*, more work is taken from the engine than is developed. This loss of work is represented by the area *C c D*. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from *q* to *r*. As the crank moves from *r* to *s*, excess energy is again developed given by the area *D d E* and the speed again increases. As the piston moves from *s* to *e*, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called ***fluctuations of energy***. The areas *BbC*, *CcD*, *DdE*, etc. represent fluctuations of energy.

A little consideration will show that the engine has a maximum speed either at *q* or at *s*. This is due to the fact that the flywheel absorbs energy while the crank moves from *p* to *q* and from *r* to *s*. On the other hand, the engine has a minimum speed either at *p* or at *r*. The reason is that the flywheel gives out some of its energy when the crank moves from *a* to *p* and *q* to *r*. The difference between the maximum and the minimum energies is known as ***maximum fluctuation of energy***.

LECTURE-5

(13) Fluctuation of Speed:

This is defined as the ratio of the difference between the maximum and minimum angular speeds during a cycle to the mean speed of rotation of the crank shaft.

(14) Maximum fluctuation of energy:

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. The horizontal line AG represents the mean torque line. Let a_1, a_3, a_5 be the areas above the mean torque line and a_2, a_4 and a_6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

Let the energy in the flywheel at $A = E$, then from Fig. we have

Energy at $B = E + a_1$

Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$

Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

= Energy at A (*i.e.* cycle repeats after G)

Let us now suppose that the greatest of these energies is at B and least at E. Therefore,

A flywheel stores energy when the supply is in excess and releases energy when energy is in deficit.

Maximum energy in flywheel

$$= E + a_1$$

Minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

∴ Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

LECTURE-6

(15) Coefficient of fluctuation of energy:

It may be defined as the **ratio of the maximum fluctuation of energy to the work done per cycle.** Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The work done per cycle (in N-m or joules) may be obtained by using the following two relations :

1. Work done per cycle $= T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned (in radians), in one revolution.

= 2π, in case of steam engine and two stroke internal combustion engines

= 4π, in case of four stroke internal combustion engines.

The mean torque (T_{mean}) in N-m may be obtained by using the following relation :

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s = $2\pi N/60$

2. The work done per cycle may also be obtained by using the following relation :

$$\text{Work done per cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute,

= N, in case of steam engines and two stroke internal combustion engines,

= $N/2$, in case of four stroke internal combustion engines.

(16) Coefficient of fluctuation of speed:

The difference between the maximum and minimum speeds during a cycle is called the *maximum fluctuation of speed*. The ratio of the maximum fluctuation of speed to the mean speed is called the *coefficient of fluctuation of speed*.

Let N_1 and N_2 = Maximum and minimum speeds in r.p.m. during the cycle, and

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2}$$

∴ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \qquad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \qquad \text{...(In terms of linear speeds)}$$

(17) Energy stored in flywheel:

A flywheel is a rotating mass that is used as an energy reservoir in a machine. It absorbs energy in the form of kinetic energy, during those periods of crank rotation when actual turning moment is greater than the resisting moment and release energy, by way of parting with some of its K.E, when the actual turning moment is less than the resisting moment.

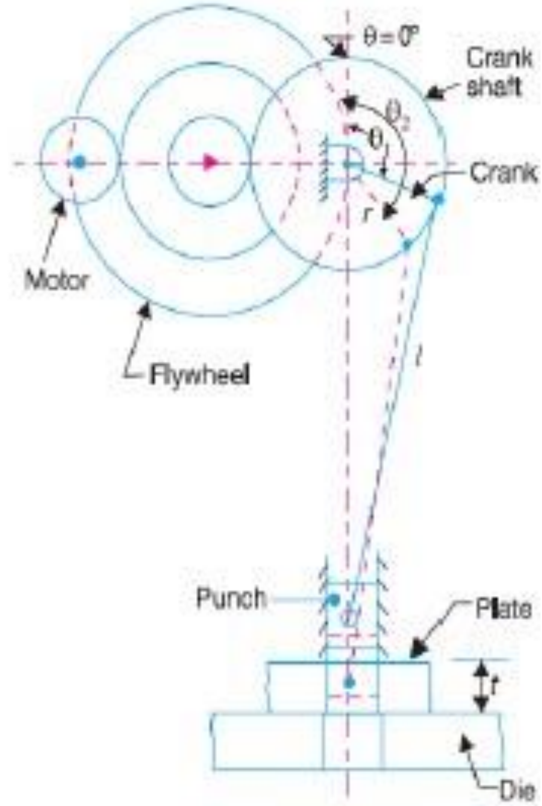
(18) Flywheel in punching press:

The flywheels used for prime movers constitute a class of problems in which the resisting torque is assumed to be constant and the driving torque varies. flywheels used in punching, riveting and similar machines constitute another class of problems in which the actual(driving) turning moment provided by an electric motor is more or less constant but the resisting torque(load) varies.

LECTURE-7

We have discussed that the function of a flywheel in an engine is to reduce the fluctuations of speed, when the load on the crankshaft is constant and the input torque varies during the cycle. The flywheel can also be used to perform the same function when the torque is constant and the load varies during the cycle. Such an application is found in punching press or in a rivetting machine. A punching press is shown diagrammatically in Fig.

The crank is driven by a motor which supplies constant torque and the punch is at the position of the slider in a slider-crank mechanism. From Fig. we see that the load acts only during the rotation of the crank from $\theta = \theta_1$ to $\theta = \theta_2$, when the actual punching takes place and the load is zero for the rest of the cycle. Unless a flywheel is used, the speed of the crankshaft will increase too much during the rotation of crankshaft will increase too much during the rotation of crank from $\theta = \theta_2$ to $\theta = 2\pi$ or $\theta = 0$ and again from $\theta = 0$ to $\theta = \theta_1$, because there is no load while input energy continues to be supplied. On the other hand, the drop in speed of the crankshaft is very large during the rotation of crank from



Operation of flywheel in a punching press.

LECTURE-8

$\theta = \theta_1$ to $\theta = \theta_2$ due to much more load than the energy supplied. Thus the flywheel has to absorb excess energy available at one stage and has to make up the deficient energy at the other stage to keep to fluctuations of speed within permissible limits. This is done by choosing the suitable moment of inertia of the flywheel.

Let E_1 be the energy required for punching a hole. This energy is determined by the size of the hole punched, the thickness of the material and the physical properties of the material.

Punching press and flywheel.

Let d_1 = Diameter of the hole punched,

t_1 = Thickness of the plate, and

τ_u = Ultimate shear stress for the plate material.

$\therefore$ Maximum shear force required for punching,

$$F_S = \text{Area sheared} \times \text{Ultimate shear stress} = \pi d_1 . t_1 \tau_u$$

It is assumed that as the hole is punched, the shear force decreases uniformly from maximum value to zero.

$\therefore$ Work done or energy required for punching a hole,