NPR COLLEGE OF ENGINEERING & TECHNOLOGY

Subject : DYNAMICS OF MACHINERY

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NPRCET

UNIT 3

FREE VIBRATION

DYNAMICS OF MACHINERY

BASIC FEATURES OF VIBRATING SYSTEMS

Define Vibration. Mention the energy conversion in vibration.

Any motion that exactly repeats itself after an interval of time is called vibration. Hence vibration is a periodic motion.

At mean or equilibrium position, strain or potential energy is converted to kinetic energy.

At extreme position, kinetic energy is converted to strain or potential energy.

Mention any two causes of vibration.

(a) Unbalance forces in the machine. These forces are produced from within the machine itself. To avoid vibrations, these forces must be balanced.

(b) External excitations: These excitations may be periodic or random.

What are the types of vibratory motion?

Free Vibration		Forced Vibration	
Unpdamed	Damped	Undamped	Damped

DYNAMICS OF MACHINERY

BASIC FEATURES OF VIBRATING SYSTEMS

Define time period and frequency.

(a) Time period is (T) is the time taken by a vibrating body to repeat the motion itself. Unit of time period is seconds.

(b) Frequency (f) is the number of cycles completed in one second. Unit of frequency is Hz or cps.

Define Free Vibration.

When a body is allowed to vibrate on its own, after giving it an initial displacement (disturbance) then the ensuing vibration is known as ***free or natural vibration.***

Ex: Turning fork, Guitar strings. No external force is applied on the body. Frequency of free vibration is called ***free or natural frequency*** **(f_n).**

DYNAMICS OF MACHINERY

BASIC FEATURES OF VIBRATING SYSTEMS

Define Forced Vibration.

When a body vibrates under the influence of an external force, then the ensuing vibration is known as forced vibration. When an external force is applied, the body does not vibrate with its own natural frequency, but vibrates with the same frequency of the applied external force. The external force is a periodic disturbing force created by unbalance. Ex: Vibrations due to rotating and reciprocating masses of IC engines.

What is resonance?

When the frequency of external force coincides with the natural frequency of the vibrating system, then the vibration takes place with very high amplitude. This phenomenon is called resonance. Failures of major structures like buildings, bridges, airplane wings are mainly due to resonance.

DYNAMICS OF MACHINERY

BASIC FEATURES OF VIBRATING SYSTEMS

Define damped vibration.

When the amplitude of vibrations reduces in every cycle, then the vibration is known as damped vibration. This is because some amount energy possessed by the vibrating system is dissipated (wasted) in overcoming the frictional resistances to the motion. Damping is provided by connecting ‘ dashpot’. When the dashpot is connected with free vibrating body to control vibrations, it is called free damped vibrations. When the dashpot is connected with forced vibrating body to control vibrations, it is called forced damped vibrations. The frequency of damped vibrations is called damped natural frequency $f_{d.}$

What are the different types of vibration?

(a) Longitudinal vibration;

(b) Transverse vibration; and

(c) Torsional vibration.

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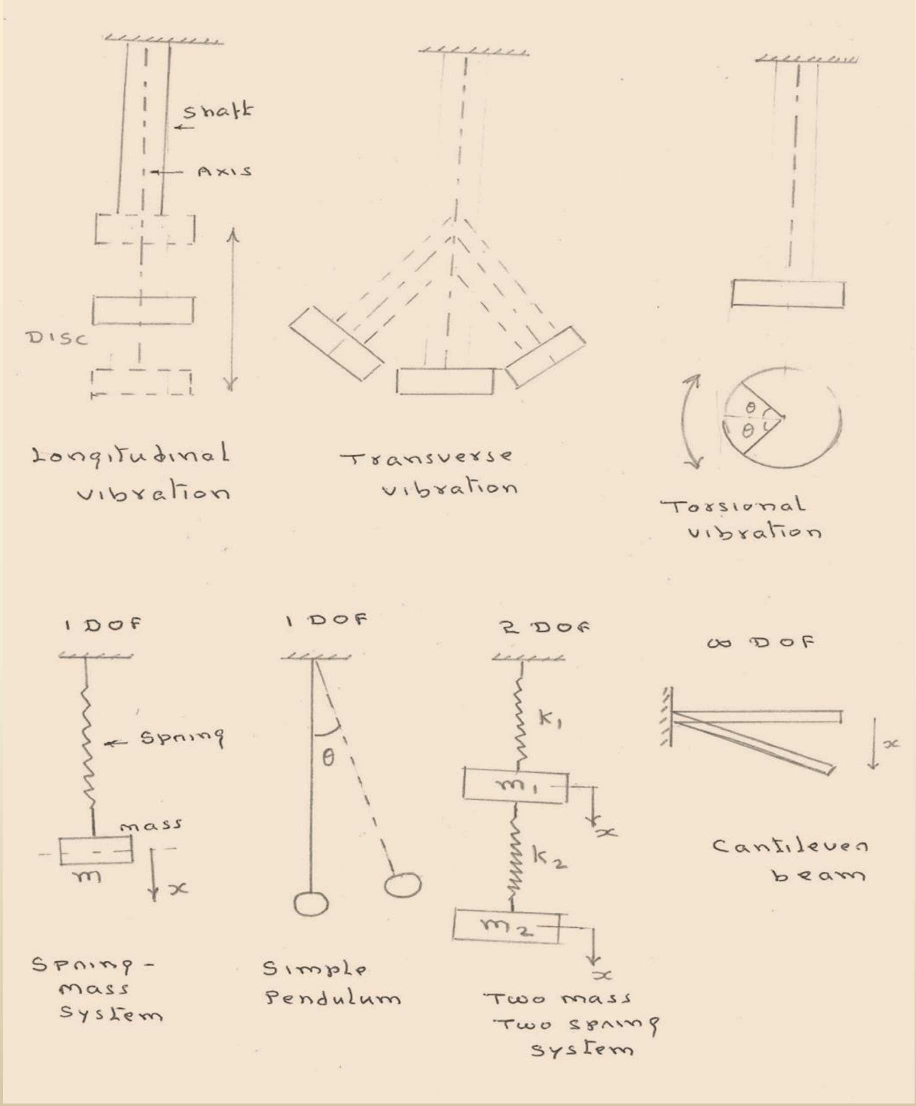
BASIC FEATURES OF VIBRATING SYSTEMS



DYNAMICS OF MACHINERY

BASIC FEATURES OF VIBRATING SYSTEMS

Which parameter brings the body to equilibrium position in rectilinear and torsional vibrations?

In longitudinal and transverse vibrations, restoring force (due to stiffness of shaft) brings the body to equilibrium position.

In torsional vibrations, restoring torque (due to torsional stiffness of shaft) brings the body to equilibrium position.

Define degrees of freedom.

The number of independent coordinates required to specify completely the configuration of a system at any instant is called degrees of freedom.

What is the difference between a discrete and continuous system?

System with finite number of degrees of freedom are called discrete or lumped parameter systems. Systems with infinite number of degrees of freedom are called continuous or distributed systems. Continuous systems are approximated as discrete systems to obtain solutions easily.

DYNAMICS OF MACHINERY

BASIC FEATURES OF VIBRATING SYSTEMS

Give two examples for bad and good effects of vibration.

(a)Bad effects: The presence of vibration in any mechanical system produces unwanted noise, high stresses, poor reliability, wear and premature failure of one or more of the parts. Vibrations also cause human discomfort in the form of physical and mental strains.

(b)Good effects: Vibration does useful work in musical instruments, vibrating screens, shakers etc. Vibration is used to relieve pain in physiotherapy.

Explain longitudinal vibration.

When the particles of the shaft or disc move parallel to the axis of the shaft, then the ensuing vibrations are called longitudinal vibrations. In this vibration, the shaft elongates and contracts alternately. Hence tensile stress and compressive stress are induced in the shaft alternately.

DYNAMICS OF MACHINERY

BASIC FEATURES OF VIBRATING SYSTEMS

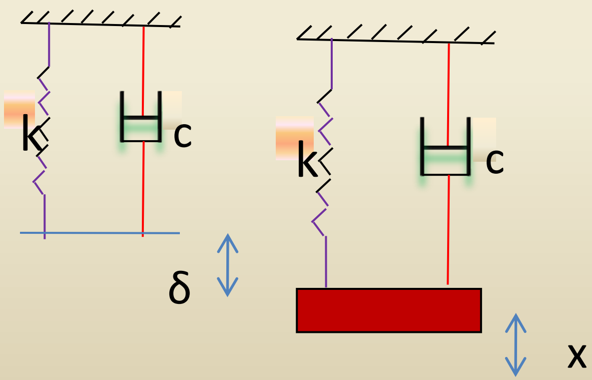
Represent by a sketch free damped longitudinal vibration.

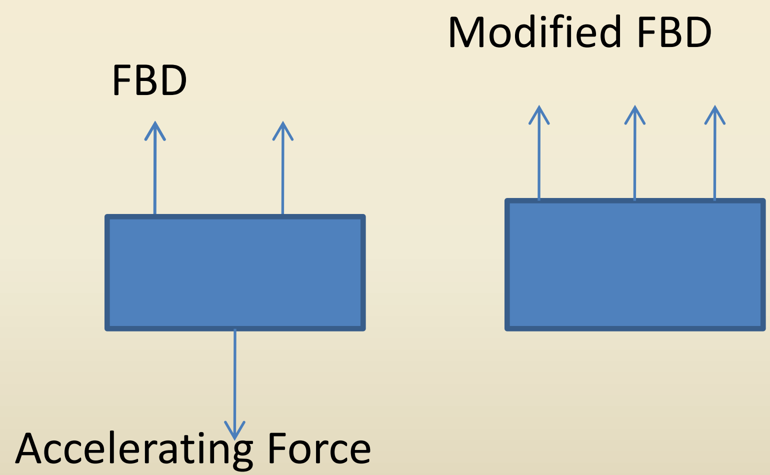
K is stiffness of the constraint in N/m

‘ c is damping coefficient in N/(m/s) [It is the damping force per unit velocity]

‘ m is the mass of the body suspended from the constraint in kg.

In the modified FBD, spring force, damping force and inertia force are shown.





DYNAMICS OF MACHINERY

BASIC FEATURES OF VIBRATING SYSTEMS

Define stiffness of spring.

It is the force required to produce unit displacement in the direction of vibration. It is expressed in N/m.

Mention the governing equation of free undamped longitudinal vibration.

The governing equation of free undamped longitudinal vibration is given by,

$$m \ddot{x} + k x = 0$$

Find the equivalent stiffness of two springs connected in parallel.

Absolute static deflection of each spring = Total static deflection of the system

Deflection , $\delta_1 = \delta_2 = \delta$; $W = mg = W_1 + W_1 = k_1 \delta_1 + k_2 \delta_2 = k \delta$

Therefore , Equivalent stiffness , $k = k_1 + k_2$

DYNAMICS OF MACHINERY

BASIC FEATURES OF VIBRATING SYSTEMS

Mention the formula for natural frequency of free longitudinal vibration (i) by neglecting the effect of inertia of the constraint and (ii) by considering the effect of inertia of the constraint

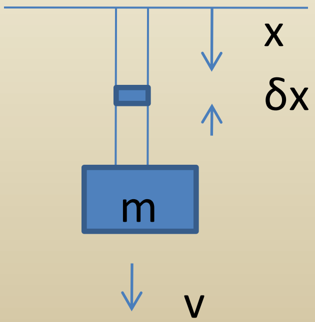
(i) Natural frequency of free longitudinal vibration by neglecting the effect of inertia of the constraint

‘ $f_n = [1/2\pi] \sqrt{(k/m)}$

(ii) Natural frequency of free longitudinal vibration by considering the effect of inertia of the constraint

‘ $f_n = [1/2\pi] \sqrt{(k/m^*)}$;

where $m^* = m + m_c$; m is mass of the suspended body and m_c is mass of the constraint.



DYNAMICS OF MACHINERY

BASIC ELEMENTS AND LUMPING OF PARAMETERS

Mention the natural frequency of oscillation of a liquid in U-tube.

‘ $f_n = [1/2\pi]\ \sqrt{(2g/l)}$ Hz; where l is length of the liquid column.

Define damping. Mention the types.

Any resistance to vibratory motion is damping. In damped vibrations, the amplitude of damping decreases over every cycle with respect to the amount of damping.

Types of damping: (a) Viscous damping ; (ii) Dry friction or coulomb damping; (iii) Solid or structural or hysteresis damping and (iv) slip or interfacial damping.

Define hysteresis damping.

Damping due to internal friction of the molecules is called solid or structural damping. This damping occurs in all vibrating systems subject to elastic restoring forces. The amount of damping is small.

DYNAMICS OF MACHINERY

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BASIC ELEMENTS AND LUMPING OF PARAMETERS

What is interfacial damping?

Damping due to microscopic slip is called slip or interfacial damping. Energy of vibration is dissipated by microscopic slip on the interfaces of machine parts in contact under fluctuating loads. Microscopic slip also occurs at joints of the machine elements.

What is the advantage of damping?

In mechanical systems damping is provided by to control the amplitude of vibration so that failure due to resonance can be avoided.

Define degree of dampness.

The ratio of $(c/2m)^2$ to (k/m) is called degree of dampness.

$DOD = [\, c / (2\, m\, \omega_n)]^2$; $\omega_n^{\,2} = k/m$; $Cc = 2m\, \omega_n$

$\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{k}{m}\right)x = 0$ Aux.equn. is $s^2 + \left(\frac{c}{m}\right)s + \left(\frac{k}{m}\right)x = 0$

$$s = \frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)}$$

DYNAMICS OF MACHINERY

BASIC ELEMENTS AND LUMPING OF PARAMETERS

Define damping coefficient.

Damping force per unit velocity is called damping coefficient.

, c = Damping force / Velocity = $F_d / \dot{x}$

Define damping factor.

Damping factor is the ratio of actual damping coefficient to critical damping coefficient.

, $\xi = c / c_c$ = Actual damping coefficient / Critical damping coefficient

The needles of an electric meter are critically damped. Why?

The needles of an electric meter are critically damped so that they can return to its original position immediately after the reading is taken.

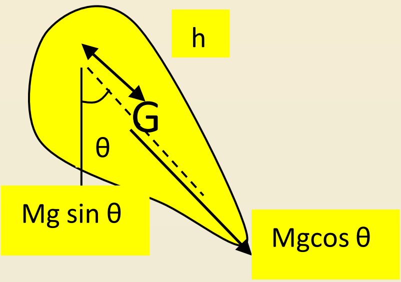
DYNAMICS OF MACHINERY

BASIC ELEMENTS AND LUMPING OF PARAMETERS

Define logarithmic decrement.

Logarithmic decrement is the natural logarithm of ratio of any two successive amplitudes of an under damped system. It is a dimensionless quantity and denoted by δ

$$\delta = \frac{2\pi\xi}{\sqrt{(1-\xi^2)}}$$



Write the expression for natural frequency of a simple pendulum.

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\,\frac{\sqrt{g}}{\sqrt{l}}$$

Write the expression for natural frequency of a compound pendulum (i.e. a rigid body suspended vertically and oscillates with small amplitude under gravity).

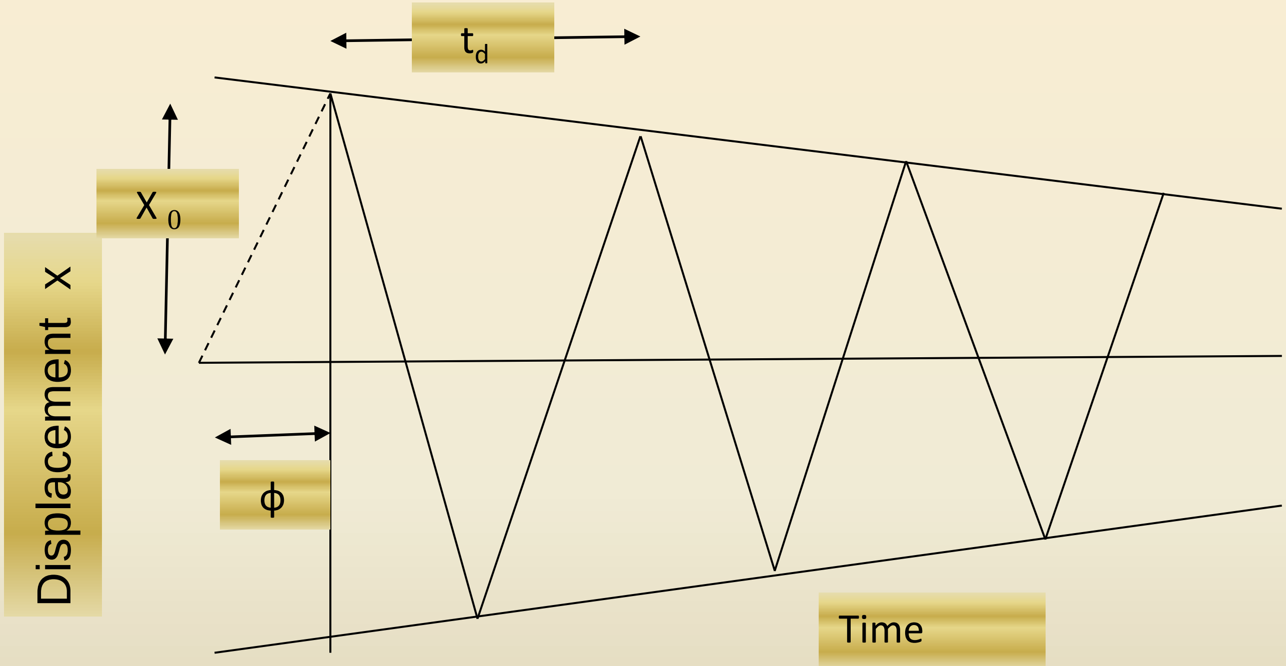
$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\,\frac{\sqrt{gh}}{\sqrt{(k^2+h^2)}}$$

‘h’ is the distance between point of suspension and CG ‘G’ of the body; ‘k’ is radius of gyration about the axis througg G perpendicular to plane of motion.

DYNAMICS OF MACHINERY

BASIC ELEMENTS AND LUMPING OF PARAMETERS

Draw the displacement – time plot of an under damped system.



Displacement vs Time Plot for under damped system

DYNAMICS OF MACHINERY

BASIC ELEMENTS AND LUMPING OF PARAMETERS

Mention the methods used to determine the natural frequency of longitudinal vibration.

(i) Equilibrium method: [D'Alembert's principle – Sum of all forces including inertia is zero]

(ii) Energy method : [Total energy is constant at all times $\frac{d(KE+PE)}{dt} = 0$]

(iii) Rayleigh's method.[Max. KE at mean position = Max. PE at extreme position]

$$\ddot{x} + \omega_n{}^2\, x = 0$$

$$\frac{d\left[\left(\frac{1}{2}\right) m \left(\frac{dx}{dt}\right)^2 + \left(\frac{1}{2}\right) kx^2\right]}{dt}$$

$$\frac{1}{2}\, m\, v_{max}{}^2 = \frac{1}{2}\, kx^2$$

DYNAMICS OF MACHINERY

TRANSVERSE VIBRATION

Define transverse vibration.

When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft (constraint), then the ensuing vibrations are called transverse vibrations. In this vibration, the shaft bends and straightens alternately. Hence bending stresses are induced in the shaft. Ex: Vibrations of beams (SSB, Cantilever)

Mention the formula for natural frequency of transverse vibration of a simply supported beam with point load at the centre taking into account the effect of inertia of the constraint.

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;
;

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\,\frac{\sqrt{k}}{\sqrt{(m + (17/35)m_c)}}$$

DYNAMICS OF MACHINERY

TRANSVERSE VIBRATION

Mention the formula for the static deflection of a simply supported beam with (a) eccentric load ; b) udl

, eccentric load $$\delta = \frac{W a^2 b^2}{3 E I L}$$

, uniformly distributed load, $$\delta = \frac{5 w L^4}{384 E I}$$

DYNAMICS OF MACHINERY

TRANSVERSE VIBRATION

Name the methods used to find the natural frequency of transverse vibration for a simply supported beam with several point loads.

(a) Energy method: This method gives accurate results, but involves tough calculations for many loads.

(b) Dunkerly's method: This method is semi empirical and simple, but gives approximate results.

Briefly explain Dunkerly's method used in natural frequency of transverse vibration.

Dunkerly's empirical formula for the natural frequency of transverse vibrations for a shaft with number of loads is

$$f_n = \frac{0.4985}{\sqrt{[\delta_1 + \delta_2 + \delta_3 + \delta_4 + \ldots\ldots + (\delta_u / 1.37)]}}$$

Where $\delta_1, \delta_2, \delta_3, \delta_4 \ldots\ldots$ are static deflection due to point loads W_1, W_2, W_3, W_4 and δ_u is static deflection at mid span due to udl.

DYNAMICS OF MACHINERY

TRANSVERSE VIBRATION

Briefly explain Rayleigh's principle used to find natural frequency of transverse vibration.

In extreme positions: Shaft possesses maximum potential energy and no kinetic energy.

In the mean position: Shaft possesses maximum kinetic energy and no potential energy.

Maximum potential energy at extreme position

= Maximum kinetic energy at mean position.

$$\frac{1}{2} m \, v_{max}^2 = \frac{1}{2} k x^2$$

$$v_{max} = \omega_n \, x \qquad \omega_n = \sqrt{\frac{k}{m}}$$

DYNAMICS OF MACHINERY

TRANSVERSE VIBRATION

Define critical speed of a shaft.

Critical speed or whirling speed of a shaft is the speed of a rotating shaft at which the shaft tends to vibrate violently in the transverse direction. It is dangerous to continue to run the shaft at its critical speed as the amplitude of vibrations will be very high and the system may go to pieces.

What are the factors affecting critical speed of a shaft?

(a) Eccentricity (distance between CG of rotor and axis of rotation;

(b) Rotational speed of the rotor.

What is the value of additional deflection of a shaft at resonance? Explain.

Additional deflection, $y = \dfrac{e}{(\omega_n / \omega)^2 - 1}$

When $\omega = \omega_n$, deflection is infinitely large due to resonance.

DYNAMICS OF MACHINERY

TORSIONAL VIBRATION

Define torsional vibration.

When a shaft is rotated from equilibrium position to extreme position, then released, the particles of the shaft or disc move in a circle about the axis of the shaft. The ensuing vibrations are called torsional vibrations.

Define torsional stiffness of a shaft.

Torque required per unit angular displacement of the shaft is called torsional stiffness of shaft.

,
,

$$q = \frac{T}{\theta} = \frac{C\,J}{L} \text{ in Nm / rad}$$

T is torsion in Nm; θ is angular displacement in radians; C is modulus of rigidity in N/m^2 ; J is polar moment of inertia in m^4; L is length of the shaft in m.

$$J = \frac{\pi}{32} d^4$$

Where d is diameter of shaft in m

DYNAMICS OF MACHINERY

TORSIONAL VIBRATION

Differentiate between rectilinear and torsional vibrations.

S.No	Rectilinear vibration	Torsional vibration
1	In longitudinal and transverse vibrations, restoring force (due to stiffness of shaft) brings the body to equilibrium.	In torsional vibrations, restoring torque (due to torsional stiffness of shaft) brings the body to equilibrium.
2	Restoring force is directly proportional to the linear displacement of mass from its equilibrium position.	Restoring torque is directly proportional to angular displacement of the mass from its equilibrium position.

DYNAMICS OF MACHINERY

TORSIONAL VIBRATION

Mention the expression for natural frequency of free torsional vibration (a) without considering the effect of inertia of the constraint; (b) considering the effect of inertia of the constraint.

Natural frequency of free torsional vibration without considering the effect of inertia of the constraint,

$$f_n = \frac{1}{T} = \frac{1}{2\pi} \frac{\sqrt{q}}{\sqrt{I}}$$

Natural frequency of free torsional vibration considering the effect of inertia of the constraint,

$$f_n = \frac{1}{T} = \frac{1}{2\pi} \frac{\sqrt{q}}{\sqrt{(I + (I_c/3)}} \text{ Hz.}$$

I_c is mass moment of inertia of constraint.

DYNAMICS OF MACHINERY

TORSIONAL VIBRATION

Define node and anti node in torsional equivalent shaft.

The point or section of the shaft whose amplitude of vibration is zero is known as node.

The point or section of the shaft whose amplitude of vibration is maximum is known as anti node.

Define torsionally equivalent shaft.

A torsionally equivalent shaft is one which has the same torsional stiffness as that of the actual shaft. It twists to the same extent under a given torque as that of the actual shaft. Length of torsionally equivalent shaft is given by

$$l_{\text{eq}} = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4 + l_3 \left(\frac{d_1}{d_3}\right)^4 + \cdots$$

DYNAMICS OF MACHINERY

TORSIONAL VIBRATION

Mention the expression for the frequency of vibration of a two rotor system.

Natural frequency of vibration of a two rotor system is ,

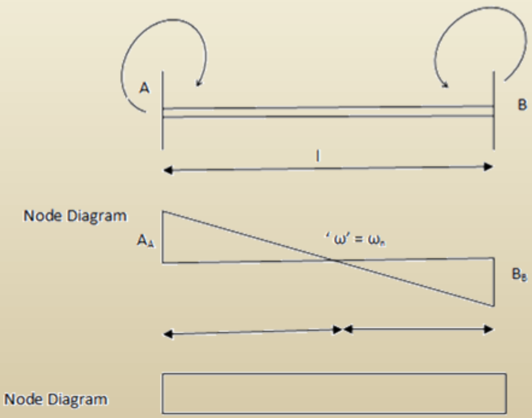
‘ $f_{n\,a} = (1/2\pi)\sqrt{[CJ/I_a L_a]}$; $f_{n\,b} = (1/2\pi)\sqrt{[CJ/I_b L_b]}$; where a and b are rotors

In case of a torsional vibration, the node of a two rotor system will be situated closer to the rotor having large mass moment of inertia. - Justify.

We know for a two rotor system,

$I_a L_a = I_b L_b$; $L_a = I_b L_b / I_a$;

In the above expression for L_a , if $I_a > I_b$, then $L_a < L_b$; hence the result.



A_A and B_B are amplitudes

DYNAMICS OF MACHINERY

TORSIONAL VIBRATION

Obtain the equations used to determine the position of node in two rotor systems.

$$L = L_a + L_b \quad \ldots\ldots\ldots (1)$$

$$L_a = I_b L_b / I_a \quad \ldots\ldots (2)$$

L_b is obtained using the above two equations. Then L_a is obtained using ..(1)

Mention the expression for the length of torsionally equivalent shaft.

Length of torsionally equivalent shaft, $L = L_1 (d / d_1)^4 + L_2 (d / d_2)^4 + L_3 (d / d_3)^4 + \ldots\ldots$

Mention the conditions required to replace a geared system by an equivalent system.

(a) Kinetic energy of the equivalent system must be equal to that of the original system.

(b) Strain energy of the equivalent system must be equal to that of the original system.

DYNAMICS OF MACHINERY

GEARED SYSTEM

Mention the conditions required to replace a geared system by an equivalent system.

(a) Kinetic energy of the equivalent system must be equal to that of the original system.

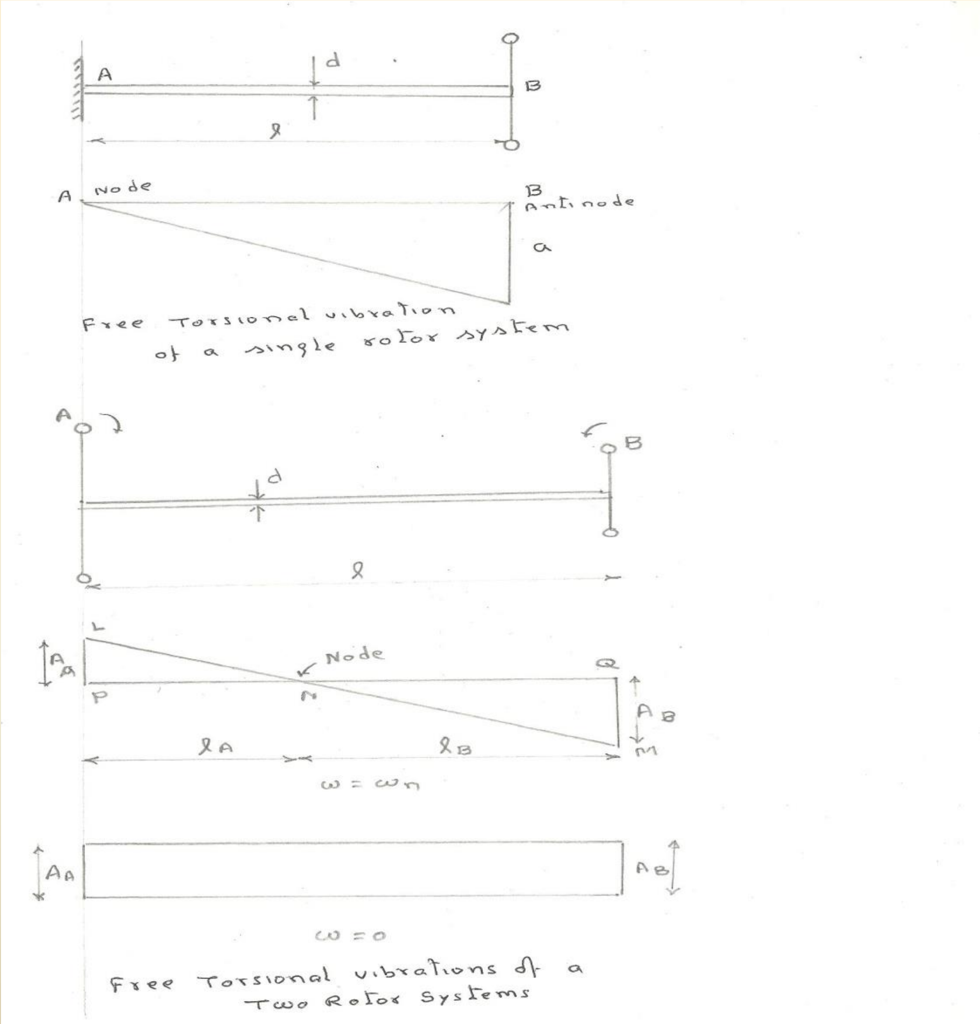
(b) Strain energy of the equivalent system must be equal to that of the original system.

What is the advantage of a geared system?

In order to increase or decrease the speed of the driven shaft, a set of geared system is connected in between driving shaft and driven shaft. Based on the value of gear ratio, the speed of the driven shaft can be varied though the driving shaft rotates at the same speed.

DYNAMICS OF MACHINERY

TORSIONAL VIBRATION



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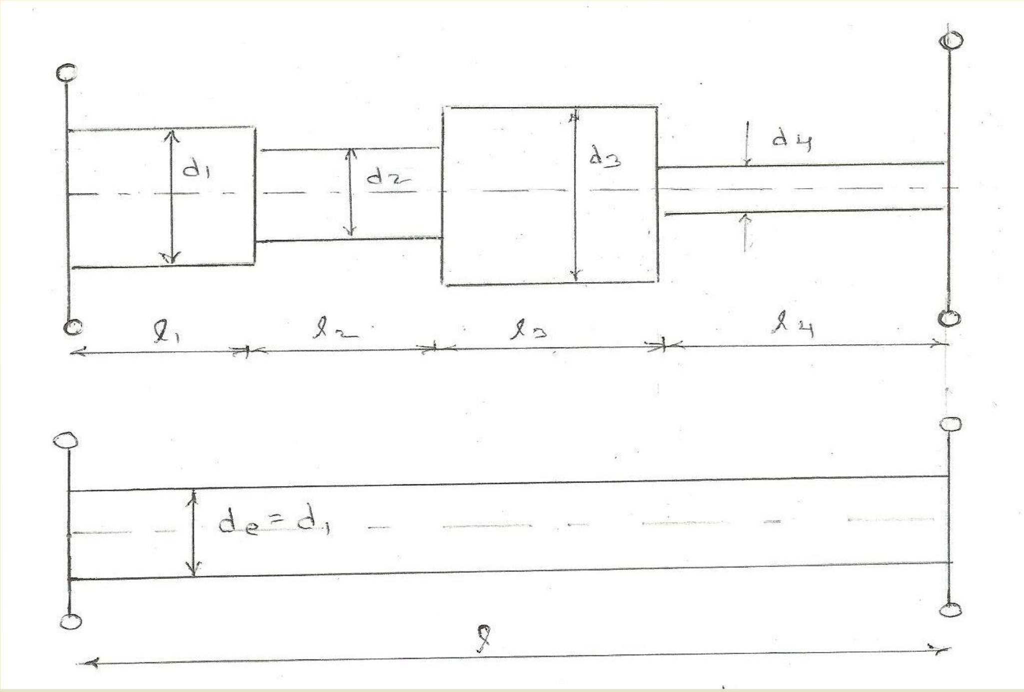
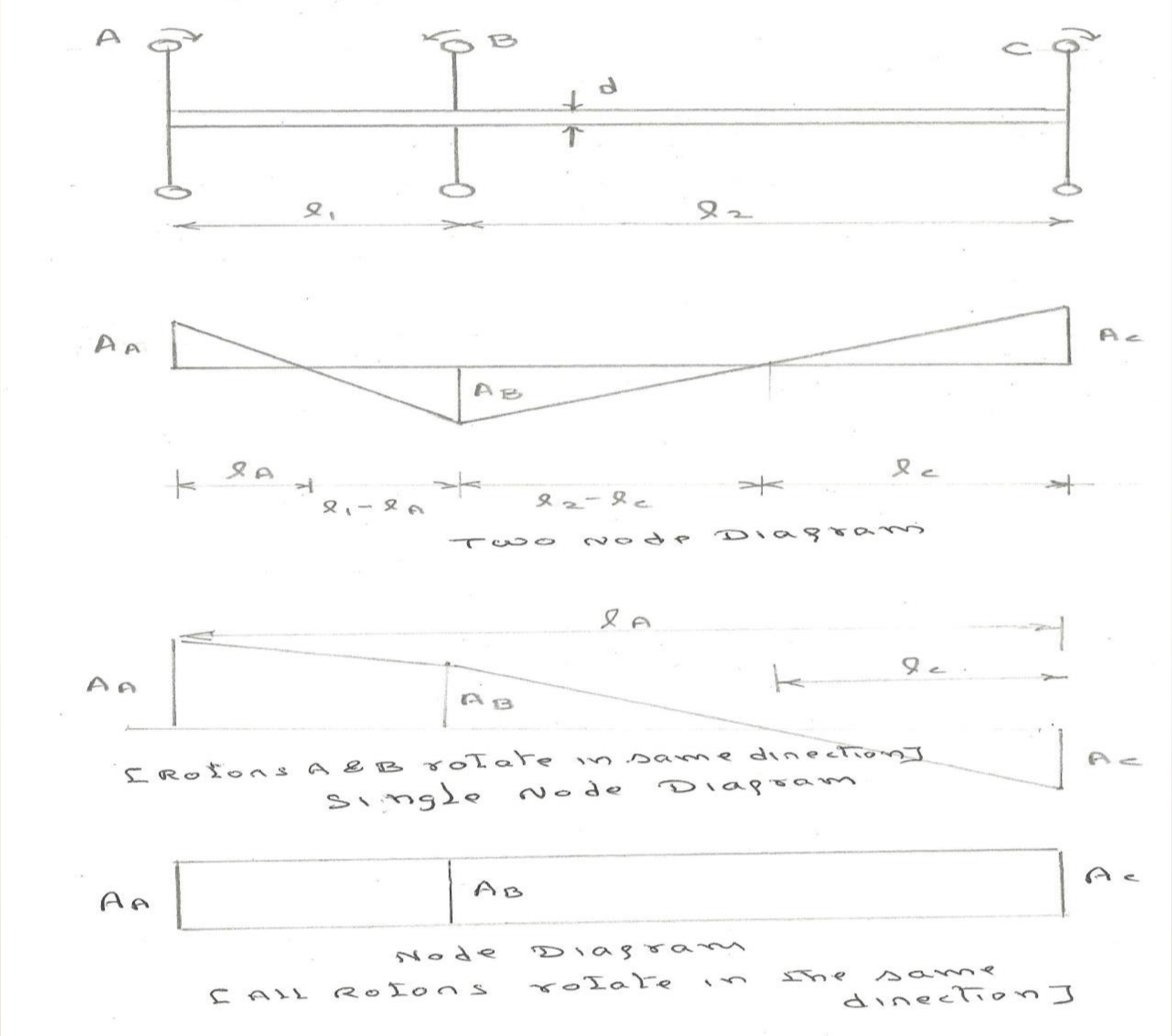
TORSIONALLY EQUIVALENT SHAFT



$$l_{eq} = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4 + l_3 \left(\frac{d_1}{d_3}\right)^4 + l_4 \left(\frac{d_1}{d_4}\right)^4$$

DYNAMICS OF MACHINERY

TORSIONAL VIBRATION



UNIT-3 Phase I

$f = \omega_n / 2\pi$; $\omega_n = \sqrt{\frac{k_e}{m}}$ (or) $\sqrt{\frac{k_e}{m + \frac{m_c}{3}}}$

$T = \frac{1}{f}$

$A = \sqrt{x_0^2 + (\dot{x}_0 / \omega_n)^2}$

$\ddot{x}_{max} = A\,\omega_n^2$

$\psi = \tan^{-1}\left[x_0 \omega_n / \dot{x}_0 \right]$

$k_e = k_1 + k_2$ for parallel springs

$1/k_e = 1/k_1 + 1/k_2$ for series springs

Static deflection

$\delta = PL / AE$ $\delta_{st} = \frac{g}{\omega_n^2}$

For cantilever $\Delta = PL^3 / 3E$

For SSB $\Delta = PL^3 / 48EI$

DYNAMICS OF MACHINERY

UNIT-3 phase II

Logarithmic decrement

$$\delta = \frac{1}{j} \ln \frac{x_1}{x_{j+1}}$$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$; j is no of cycles

ξ is damping factor

Damping coef or Damping ratio c

$$c = c_{cr}\,\xi$$

$$c_{cr} = 2\sqrt{km}$$

$$\omega_{nd} = \omega_n\sqrt{1-\xi^2}$$

$$T_d = \frac{2\pi}{\omega_d} = \frac{T_n}{\sqrt{1-\xi^2}}$$

UNIT-3 Phase I

$$f = \omega_n / 2\pi \quad ; \quad \omega_n = \sqrt{\frac{k_e}{m}} \quad (\text{or}) \quad \sqrt{\frac{k_e}{m + \frac{m_c}{3}}}$$

$$T = \frac{1}{f}$$

$$A = \sqrt{x_0^2 + (\dot{x}_0/\omega_n)^2}$$

$$\ddot{x}_{max} = A\,\omega_n^2$$

$$\psi = \tan^{-1}\left[x_0\omega_n / \dot{x}_0 \right]$$

$k_e = k_1 + k_2$ for parallel springs

$1/k_e = 1/k_1 + 1/k_2$ for series springs

Static deflection

$$\delta = PL/AE \qquad \delta_{st} = \frac{g}{\omega_n^2}$$

For cantilever $\Delta = PL^3/3E$

For SSB $\Delta = PL^3/48EI$

UNIT-3 Phase II

Logarithmic decrement

$$\delta = \cancel{x_2} \; \frac{1}{j} \ln \frac{x_1}{x_{j+i}}$$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$; j is no of cycles

ξ is damping factor

Damping coef or Damping ratio c

$$c = c_{cr}\,\xi$$

$$c_{cr} = 2\sqrt{km}$$

$$\omega_{nd} = \omega_n \sqrt{1-\xi^2}$$

$$T_d = \frac{2\pi}{\omega_d} = \frac{T_n}{\sqrt{1-\xi^2}}$$

DYNAMICS OF MACHINERY

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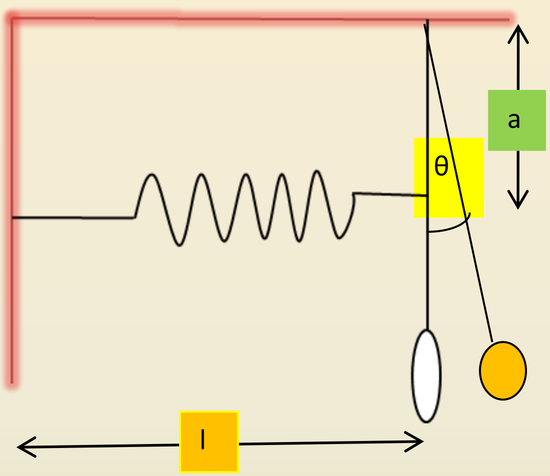
Derive the differential equation of motion for spring controlled simple pendulum

Spring Force Fs = - kaθ; Where aθ is expansion in the spring



Accelerating Torque $I_0 \frac{d^2\theta}{dt2}$

Restoring Torque $-(mg\theta l + ka\theta a)$

Equating the two, $I_0 \frac{d^2\theta}{dt2} = -(mg\theta l + ka\theta a)$

$$I_0 \frac{d^2\theta}{dt2} + mgl\theta + ka^2\theta = 0$$

$$I_0 = ml^2$$

$$\ddot{\theta} + \frac{(mgl+ka^2)}{ml^2}\,\theta = 0$$

Hence $\omega_n^2 = \frac{(mgl+ka^2)}{ml^2}$

DYNAMICS OF MACHINERY

FREE VIBRATION

Given: Shaft – Freely supported; Shaft diameter = 0.025 m; Length of shaft = 0.7 m
Load at mid point = 1 kg; Density of shaft material = 40000 kg/m^3 ;
Young's modulus E = 210 x 10 ^{9} m^2
To find: Whirling speed of the shaft

Moment of inertia for circular shaft, $I = \frac{\pi}{64} d^4$

Moment of inertia for rectangular beam, $I = \frac{b\, d^3}{12}$

For simply supported beam with point load static deflection, $\delta = \frac{W a^2 b^2}{3EIL}$

For simply supported beam with uniformly distributed load static deflection, $\delta s = \frac{5 w L^4}{384\, E\, I}$

Dunkerley's formula for natural frequency of transverse vibration for beams,

$$fn = \frac{0.4985}{\sqrt{\delta_1 + \delta_s}}$$

DYNAMICS OF MACHINERY

FREE TRANSVERSE VIBRATION

Given: Shaft – Freely supported; Shaft diameter = 0.010 m; Length of shaft = 0.4 m
Load at mid point = 12 kg; Density of shaft material = 7500 kg/m^3 ;
Young's modulus E = 200 x 10 ^{9} N/m^2
To find: Whirling speed of the shaft
Ans: I = 4.91x10-^{10} m^4; δ = 1.598x10^{-3} m; fn = 12.47 Hz; Nc = 748 rpm;
δ s= 1.962x10^{-5} m; fn = 12.41 Hz; Nc = 744.6 rpm;

Moment of inertia for circular shaft, $I = \frac{\Pi}{64} d^4$

Moment of inertia for rectangular beam, $I = \frac{b\, d^3}{12}$

For simply supported beam with point load static deflection, $\delta = \frac{W a^2 b^2}{3EIL}$

For simply supported beam with uniformly distributed load static deflection, $\delta s = \frac{5wL^4}{384\, E\, I}$

Dunkerley's formula for natural frequency of transverse vibration for beams,

$$fn = \frac{0.4985}{\sqrt{\delta_1 + \delta_s}}$$

DYNAMICS OF MACHINERY

Given: Steel bar Dimensions: 2.5 cm wide, 5 cm deep; Freely supported at 1 m apart;
Mass at mid point m = 200 kg; Weight of the bar is negligible; E = 27 x 10^{10} N/m^2
To find: (a) frequency of natural transverse vibration;
(b) Frequency of vibration if additional mass of 200 kg is distributed uniformly along the length of the shaft.

Moment of inertia for rectangular beam, $I = \frac{b\,d^3}{12}$

For simply supported beam with point load static deflection, $\delta = \frac{Wa^2b^2}{3EIL}$

For simply supported beam with uniformly distributed load static deflection, $\delta s = \frac{5wL^4}{384\,E\,I}$

Dunkerley's formula for natural frequency of transverse vibration for beams,

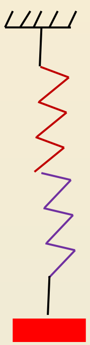
$$fn = \frac{0.4985}{\sqrt{\delta_1 + \delta_s}}$$

Ans: I = 2.6x10-^{7}m^4; δ = 58.23x10^{-5} m; fn = 20.66 Hz
δ s= 36.39x10^{-5} m; fn 16.20Hz;

DYNAMICS OF MACHINERY

Given: Spring, Upper end fixed; Mass attached to lower end m = 5 kg;
Natural period t = 0.45 s;

To find natural period if both ends of spring are fixed and m = 2.5 kg acts at midpoint.

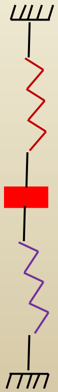


$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{t} \qquad \omega_n = \sqrt{\frac{k}{m}}$$

Let k be k_{eq} of two half springs with stiffness k_1

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_1} \qquad k_1 = 2k$$

ω_n = 13.96 rad/s ; k = 974.4 N/m



When springs are in parallel (i.e. spring is fixed at both ends),

$$k_{eq} = k_1 + k_1 = 4k$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

K eq = 3897.6 N/m; ω_n = 39.48 rad/s;

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{t}$$

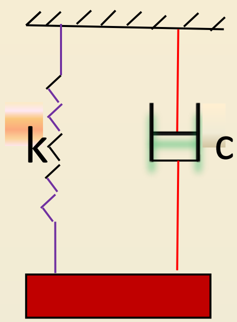
f_n = 6.28 Hz; t = 0.16 sec

DYNAMICS OF MACHINERY

EQUATIONS OF MOTION

Given: Vibrating system; Mass m = 3.5 kg; Spring stiffness k = 2.5 N/mm; Damping coefficient c = 0.018 N/mm/s

To find: (a)Damping factor; (b) Natural frequency of damped vibrations



Circular frequency of undamped vibration $$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

Critical damping coefficient $$c_c = 2m\omega_n$$

Damping factor $$\xi = \frac{c}{c_c} = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}}$$

Circular frequency of damped vibration $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

Natural frequency of damped vibration $$f_d = \frac{\omega_d}{2\pi} = \frac{1}{t_d}$$

Logrithmic decrement $$\delta = \frac{2\pi\xi}{\sqrt{(1-\xi^2)}}$$

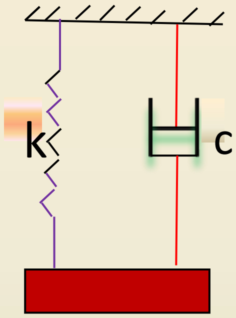
Ratio of successive amplitudes (n=1) $$\frac{x_0}{x_n} = e^{n\delta}$$

DYNAMICS OF MACHINERY

NATURAL FREQUENCY

Given: Vibrating system; Mass m = 20 kg; Spring stiffness k = 4 N/mm; Amplitude at the beginning of 4th cycle = 0.8 times previous amplitude; To find: (a) Damping factor per unit velocity; (b) Ratio of frequency of damped and undamped vibrations.

$$\omega_n = \sqrt{\frac{k}{m}} \qquad c_c = 2m\omega_n \qquad \frac{x_3}{x_4} = 1/0.8 = e^{\delta} = e^{\xi\omega_n t_d}$$



$$\xi = \frac{c}{c_c} = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}} \qquad \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_d/\omega_n = \sqrt{1-\xi^2}$$

'ω_n = 14.14 rad/s; ξ = 0.035; c_c = 565.6 Ns/m; c = 19.796 Ns/m; ω_d = 14.13 rad/s; ω_d / ω_n = 0.999

DYNAMICS OF MACHINERY

Given: damped free vibration; Mass m = 25 kg; Spring stiffness k = 15 kN/m;
Damping = 15% of critical damping;
To find: (a) Critical damping coefficient; (b) Damping factor;
(c) Natural frequency of damped vibration; (d) laogarithmic decrement;
and (e) Ratio of two consecutive amplitudes of the vibrating system

Damping factor $\xi = 0.15$ (given)

$$\xi = \frac{c}{c_c} = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}}$$

Circular frequency of undamped vibration

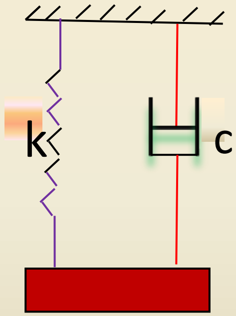
$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

Critical damping coefficient

$$c_c = 2m\omega_n$$

Circular frequency of damped vibration

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$



Natural frequency of damped vibration

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{t_d}$$

Periodic time for damped vibration

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{t_d}$$

Logarithmic decrement

$$\delta = \frac{2\pi\xi}{\sqrt{(1-\xi^2)}}$$

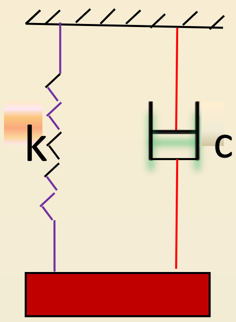
Ratio of successive amplitudes (n=1)

$$\frac{x_0}{x_n} = e^{n\delta}$$

'ω_n = 24.5 rad/s; c_c = 1225 Ns/m; c = 183.75 Ns/m; fd = 3.855 Hz;ω_d = 24.22 rad/s; δ = 0.9532; X0/X1 = 2.594

DYNAMICS OF MACHINERY

Given: vibrating system; Mass m = 3 kg; Spring stiffness k = 100 N/m; Damping coefficient, c = 3 Ns/m;
To find: (a) Damping factor; (b) Natural frequency of damped vibrationl; (c) Logarithmic decrement; (d) Ratio of two consecutive amplitudes; and (e) Number of cycles after which amplitude is reduced to 20%.



Circular frequency of undamped vibration $\omega_n = \sqrt{\frac{k_{eq}}{m}}$

Critical damping coefficient $c_c = 2m\omega_n$

Circular frequency of damped vibration $\omega_d = \omega_n\sqrt{1-\xi^2}$

Damping factor ξ $\xi = \frac{c}{c_c} = \sqrt{\frac{\delta^2}{4\pi^2+\delta^2}}$

Natural frequency of damped vibration $f_d = \frac{\omega_d}{2\pi} = \frac{1}{t_d}$

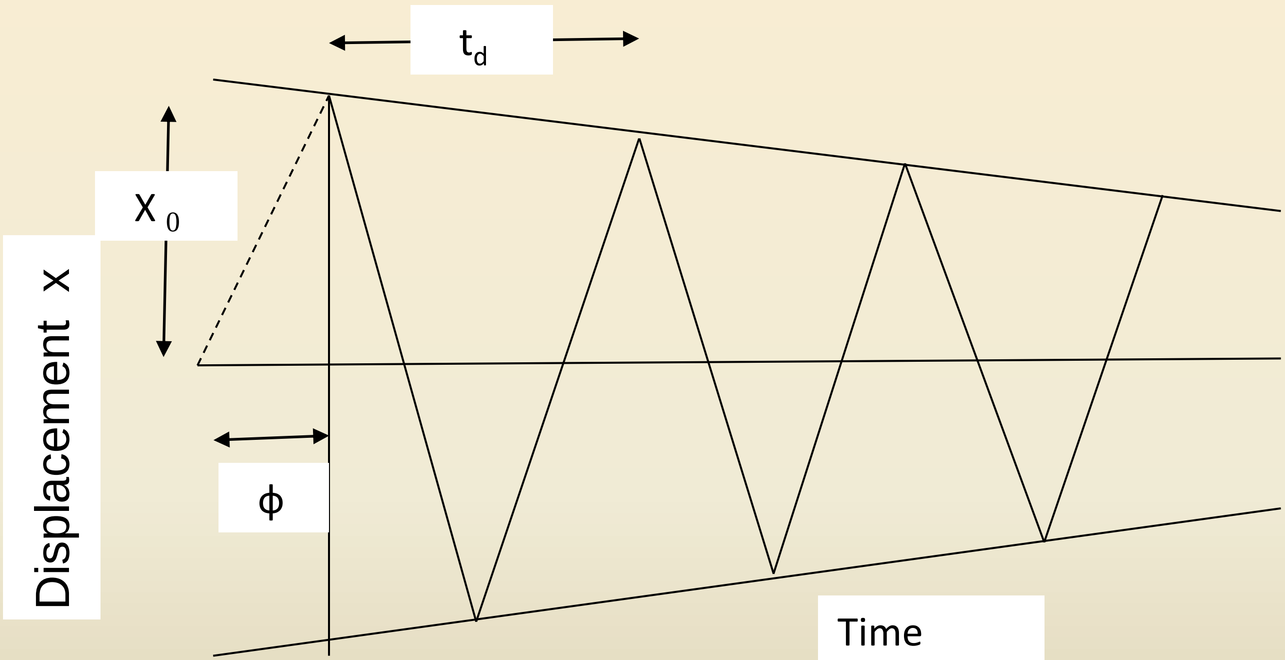
Periodic time for damped vibration

Logarithmic decrement $\delta = \frac{2\pi\xi}{\sqrt{(1-\xi^2)}}$

No. of cycles for X0/Xn = 1/0.2 $\frac{x_0}{x_n} = e^{n\delta}$

DYNAMICS OF MACHINERY

Displacement vs Time Plot for under damped system



Displacement vs Time Plot for under damped system

DYNAMICS OF MACHINERY

Given: Damped oscillations; Mass m = 7.5 kg; Time for 60 oscillations t = 35 s;
Ratio of 1st and 7th displacement = 2.5;
To find: (a) Stiffness of the spring; (b) Damping resistance c in Ns/m; and (c) Damping resistance required for critical damping.

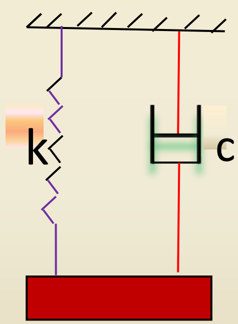
Natural frequency of damped vibration — $f_d = 60/35 = 1.714$ Hz

Circular frequency ω_d — $$f_d = \frac{\omega_d}{2\pi} = \frac{1}{t_d}$$

Logarithmic decrement δ (n=7) — $$\frac{x_0}{x_n} = e^{n\delta} = e^{n\xi\omega_n t_d}$$

Damping factor ξ — $$\delta = \frac{2\pi\xi}{\sqrt{(1-\xi^2)}}$$

Circular frequency of undamped vibration ω_n — $$\omega_d = \omega_n\sqrt{1-\xi^2}$$



Stiffness of spring k — $$\omega_n = \sqrt{\frac{k}{m}}$$

Critical damping resistance c_c — $$c_c = 2m\omega_n$$

Damping resistance c — $$\xi = \frac{c}{c_c} = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}}$$

DYNAMICS OF MACHINERY

Given: Instrument; Frequency of vibration if there is no damping = 1 Hz;
Frequency of damped vibration = 0.9 Hz;
To find: (a) Damping factor; (b) Logarithmic decrement

Given f_n = 1 Hz; f_d = 0.9 Hz

Circular frequency of undamped vibration ω_n

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{t}$$

Circular frequency of damped vibration ω_d

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{t_d}$$

Time period t_d

Damping factor ξ

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

Critical damping coefficient c_c

$$\xi = \frac{c}{c_c} = \sqrt{\frac{\delta^2}{4\pi^2+\delta^2}}$$

Ratio of successive amplitudes (n = 1), X_0/X_1

$$\frac{x_0}{x_n} = e^{n\delta} = e^{n\xi\omega_n t_d}$$

Logarithmic decrement δ

$$\delta = \frac{2\pi\xi}{\sqrt{(1-\xi^2)}}$$

DYNAMICS OF MACHINERY

TRANSVERSE VIBRATION

Given: Cantilever shaft; Shaft diameter d = 0.05 m; Length of shaft l = 0.3 m; Mass of disc at free end, m = 100 kg; Young's modulus E = 200 x 10 ^{9} N/m^2 ;
To find: (a) Frequency of longitudinal vibration; (b) Frequency of transverse vibration of the shaft.

Longitudinal vibration:

Displacement δ $\delta = \frac{WL}{A\,E}$

Area of cross section of shaft A $A = = \frac{\pi}{4} d^2$

Dunkerley's formula for natural frequency of longitudinal vibration $fn = \frac{0.4985}{\sqrt{\delta_1 + \delta_s}}$

Transverse vibration:

Moment of Inertia of shaft I $I = \frac{\pi}{64} d^4$

Static deflection for point load at the end of cantilever $\delta = \frac{WL^3}{3\,E\,I}$

Dunkerley's formula for natural frequency of transverse vibration $fn = \frac{0.4985}{\sqrt{\delta_1 + \delta_s}}$

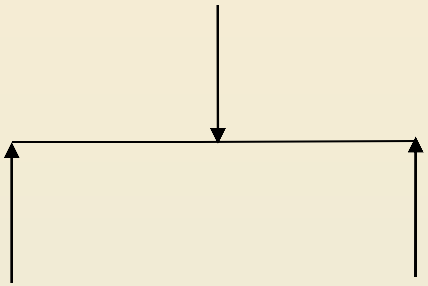
DYNAMICS OF MACHINERY

NATURAL FREQUENCY OF TRANSVERSE VIBRATION SYSTEM

Given: Simply supported shaft;
Diameter of shaft d = 0.02 m; Length of shaft l = 0.6 m;
Load at mid point m = 248.2 N;
Young's modulus of shaft material E = 200 x 10^9 N/m^2 ;
To find: Critical speed of shaft in rpm.

Moment of inertia of shaft I $$I = \frac{\pi}{64} d^4$$

Static deflection due to point loads $$\delta = \frac{Wa^2b^2}{3EIL} = \frac{wL^3}{48\ E\ I}$$



Dunkerley's formula for natural frequency of transversevibration $$fn = \frac{0.4985}{\sqrt{\delta_1 + \delta_s}}$$

Critical speed N_c $$N_c = 60\ f_n$$

Ans: 'δ' = 0.000711 m; N_c = 18.4 Hz

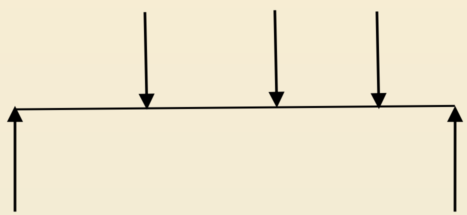
DYNAMICS OF MACHINERY

TRANSVERSE VIBRATION

Given: Simply supported shaft; Diameter d = 0.05 m; Length l = 3 m;
Young's modulus of shaft material E = 200 x 10 ^{9} N/m^2 ;

Point Loads	1000	1500	750	N
Distance from left support	1	2	2.5	m

To find: Frequency of transverse vibration



Moment of inertia of shaft I

$$I = \frac{\pi}{64} d^4$$

Static deflection due to point loads

	Load1	Load2	Load3
	1000	1500	750
'a' =	1	2	2.5
'b' =	2	1	0.5

$$\delta_1 = \frac{W_1 a^2 b^2}{3EIL}$$

$$\delta_2 = \frac{W_2 a^2 b^2}{3EIL}$$

$$\delta_3 = \frac{W_3 a^2 b^2}{3EIL}$$

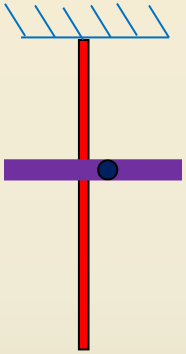
Dunkerley's formula for natural frequency of transversevibration

$$fn = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}}$$

Critical speed N_c

$$N_c = 60\, f_n$$

Given: Vertical shaft carrying a disc at centre, supported at ends;
Diameter of shaft d = 0.005 m; Length of shaft l = 0.2 m;
Mass of disc m = 50 kg;
Speed of shaft = 0.75 critical speed;
Distance of centre of disc from axis of shaft e = 0.00025 m;
Young's modulus of shaft material E = 200 x 10 ^{9} N/m^2 ;
To find: (a) Critical speed; (b) Maximum bending stress at above speed



Moment of inertia of shaft I $$I = \frac{\pi}{64} d^4$$

Static deflection $$\delta = \frac{W a^3 b^3}{3EIL^3} = \frac{WL^3}{192\ E\ I}$$

Dunkerley's formula for natural frequency of transversevibration $$fn = \frac{0.4985}{\sqrt{\delta_1 + \delta_s}}$$

Critical speed N_c $$N_c = 60\ f_n$$

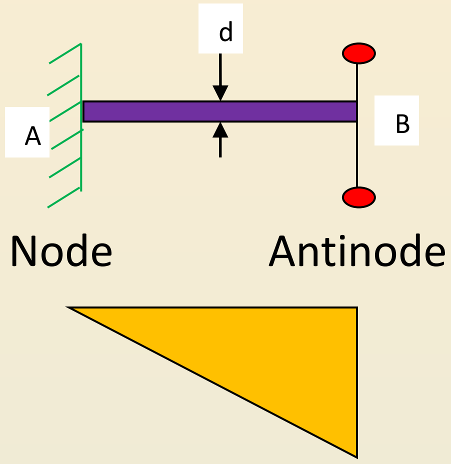
Speed of the shaft N = 0.75 Nc $$\omega = 2\pi N/60 \qquad M = m\,\omega^2\,e\,l/2$$

at speed N, Max. bending stress σ $$\frac{M}{I} = \sigma/r \qquad r = d/2$$

DYNAMICS OF MACHINERY

NATURAL FREQUENCY OF SINGLE ROTOR SYSTEM

Given: Single rotor system; Natural frequency = 5 Hz;
Diamter of steel rod d = 0.02 m; Inertia of mass fixed at free end I = 0.0098 kgm^2
Modulus of rigidity C = 0.85 x 10^{11} N/m^2
To find: Length of steel rod.



Mass moment of inertia of mass ay free end, I

$I = 0.0098 \text{ kg m}^2$

Polar momemt of inertia J

$$J = = \frac{\pi}{32} d^4$$

Natural frequency considering inertia of the constraint

$$f_n = \frac{1}{2\pi} \frac{\sqrt{q}}{\sqrt{[I + \left(\frac{Ic}{3}\right)]}}$$

Natural frequency neglecting inertia of the constraint

$$f_n = \frac{1}{2\pi} \sqrt{\frac{q}{I}} \qquad f_n = \frac{1}{2\pi} \sqrt{\frac{CJ}{IL}}$$

L = 1.32 mm

Note: q = q1 + q2 for parallel shafts; 1/q = 1/q1 + 1/q2 for series shafts;

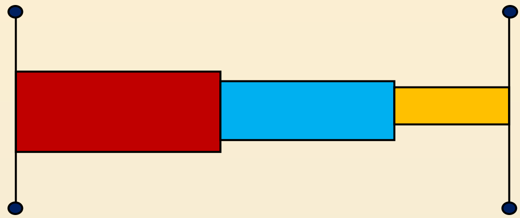
DYNAMICS OF MACHINERY

NATURAL FREQUENCY OF TWO ROTOR SYSTEM

Given: Free torsional vibration; Shaft length l = 1.5 m;

Shaft carries flywheels at two ends;

Length	0 – 0.6	0.6 – 1.1	1.1 – 1.5	m
Diameter	0.095	0.06	0.05	m

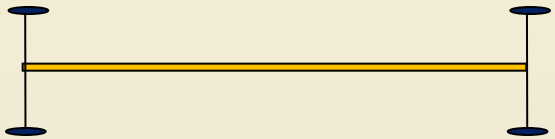


Flywheel at	0.095 dia end	0.05 dia end	
Mass	900	700	Kg
Radius of gyration	0.85	0.55	m

To find: (a)Location of node;

(b) Natural frequency of torsional vibrations

Assume Modulus of rigidity G = 80×10^{9} N/m^2 ;



$$l_{eq} = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4$$

L_{eq}= 8.97 m

$I_a = m_a\, k_a^2$ = 650.25 kgm^2

$I_b = m_b\, k_b^2$= 211.75 kgm^2

$I_a\, L_a = I_b\, L_b$

$L_a + L_b = L_{eq}$= 8.97 m

On equivlent shaft,

La = $I_b L_b / I_a$ = 2.2 m; L_b = 6.77 m

On actual shaft, $L_{a\ act} = L_1 + (L_a - L_1)\,(d_2/d_1)^4$ = 0.855 m;

Polar moment of inertia J = $$J = \frac{\pi}{32} d^4$$

q = CJ/L

Natural frequency $f_n = f_{na} = f_{nb}$ = 3.37 Hz

$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$$

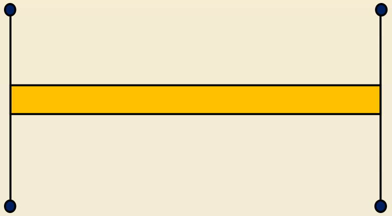
DYNAMICS OF MACHINERY

NATURAL FREQUENCY OF TWO ROTOR SYSTEM

Given: Shaft carrying flywheel and dynamo at ends;

Modulus of rigidity $C = 83 \times 10^3\ N/mm^2$

	Dynamo	Flywheel	
Mass	120	180	Kg
Radius of gyration	0.0225	0.03	m
Effective shaft dia	0.05	0.05	m
Effective length	0.2	0.25	m



To find: (a) Position of node; (b) Frequency of torsional oscillations

$$l_{eq} = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4$$

$L_{eq} = 0.45$ m

$I_a = m_a\, k_a^2 = 0.06075 kgm^2$

$I_b = m_b\, k_b^2 = 0.162\ kgm^2$

$I_a\, L_a = I_b\, L_b$

$L_a + L_b = L_{eq} = 0.45$ m

On equivlent shaft,

$La = I_b L_b / I_a = 0.327m$; $L_b = 0.123$ m

On actual shaft, $L_{a\ act} = L_1 + (L_a - L_1)\,(d_2/d_1)^4 = 0.327$ m;

$q = CJ/L$

Polar moment of inertia J = $J = \frac{\pi}{32} d^4$

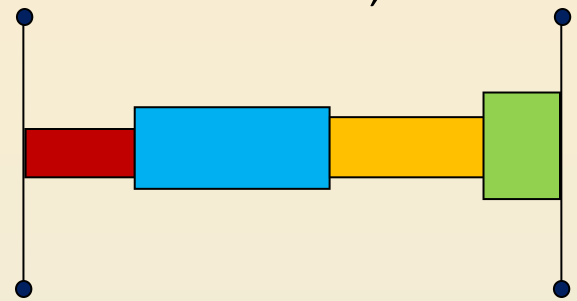
Natural frequency $f_n = f_{na} = f_{nb} =$ 254.6 Hz $f_n = \frac{1}{2\pi}\sqrt{\frac{CJ}{IL}} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_a L_a}}$

DYNAMICS OF MACHINERY

NATURAL FREQUENCY OF TWO ROTOR SYSTEM

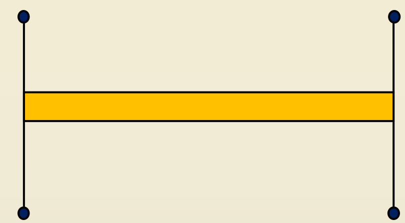
Length m	**0 – 0.275**	**0.275 – 0.775**	**0.775 – 1.15**	**1.15 – 1.25**
Dia of shaft m	0.075	0.125	0.0875	0.175

Given: Total length of shaft l = 1.25 m as above; Shaft carries two rotors at its ends;
Mass moment of inertia of Rotor 1: 75 kgm^2
Mass moment of inertia of Rotor 2: 50 kgm^2
Modulus of rigidity of shaft material C = 80 x 10^9 N/m^2
To find: Frequency of natural torsional vibrations of the shaft



$$l_{eq} = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4$$

L_{eq} = 0.545 m



$I_a = m_a k_a^2$ = 75kgm^2

$I_a L_a = I_b L_b$
$L_a + L_b = L_{eq}$ = 0.545m

$I_b = m_b k_b^2$= 50kgm^2

On equivlent shaft,
La = $I_b L_b / I_a$ = 0.218 m; L_b =0.327 m

On actual shaft, $L_{a\ act}$ = $\cancel{L_1 + (L_a - L_1)(d_2/d_1)^4}$ = 0.218 m;La < L1 hence $L_{a\ act} = L_a$

q = CJ/L

Polar moment of inertia J = $J = \frac{\pi}{32} d^4$

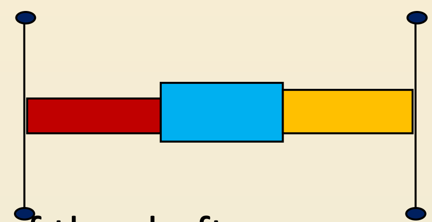
Natural frequency $f_n = f_{na} = f_{nb}$ = 19.60 Hz $f_n = \frac{1}{2\pi}\sqrt{\frac{CJ}{IL}} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_a L_a}}$

DYNAMICS OF MACHINERY

NATURAL FREQUENCY OF TWO ROTOR SYSTEM

Given: two rotor systems; Length of shaft l = 0.5 m;

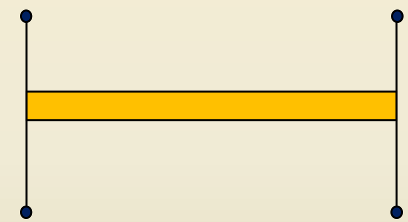
	A	B	
Mass	300	500	Kg
Radius of gyration	0.3	0.45	m



Shaft diameter 0.07 m upto 0.25 m long;
0.12 m for next 0.07 m long;
0.10 m for remaining length

Modulus of rigidity C = 80 x 10 ^{9} N/m^{2}

To find: (a) Position of node; (b) Frequency of torsional vibrations of the shaft.



$$l_{eq} = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4 + l_3 \left(\frac{d_1}{d_3}\right)^4$$

L_{eq} 0.301m

$I_a = m_a k_a^2 = 27 kgm^2$

$I_a L_a = I_b L_b$

$L_a + L_b = L_{eq} = 0.301$ m

$I_b = m_b k_b^2 = 101.25 kgm^2$

On equivlent shaft,

$La = I_b L_b / I_a =$ 0.238 m; $L_b = 0.063$ m

On actual shaft, $L_{a\ act} = L_1 + (L_a - L_1)(d_2/d_1)^4 = 0.145$ m;

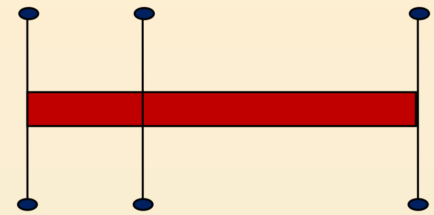
q = CJ/L

Polar moment of inertia J = $J = \frac{\pi}{32} d^4$

Natural frequency $f_n = f_{na} = f_{nb} = 27.27$ Hz $f_n = \frac{1}{2\pi}\sqrt{\frac{CJ}{IL}} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_a L_a}}$

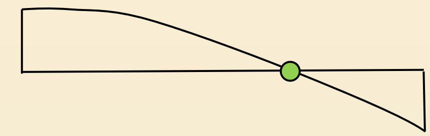
DYNAMICS OF MACHINERY

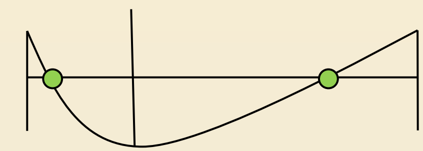
NATURAL FREQUENCY OF THREE ROTOR SYSTEM



Given: Three rotors A, B, C; Shaft diameter d = 0.04 m;
Modulus of rigidity C = 85x 10 ^{9} N/m^2
Distance between A and B = 1.25 m;
Distance between B and C = 4 m

	A	B	C	
Moment of Inertia	7.5	22.5	12.5	kgm^2





To find (a) Frequency of torsional vibrations; (b) position of nodes; (c) Amplitudes of vibrations

I_A = 7.5 kgm^2; I_B =22.5 kgm^2; I_C = 12.5 kgm^2; d = 0.04 m; C = 8.5E10 N/m^2 ; L_1 = 1.25 m; L_2 = 4 m;

In a three rotor system, $f_{nA} = f_{nB} = f_{nC}$; $I_a l_a = I_b l_b = I_c l_c$

$$l_a = \frac{I_c}{I_a} l_c \quad \text{.....(1)}$$

$$l_{eq} = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4$$

$$\frac{1}{I_c l_c} = \frac{1}{I_b}\left(\frac{1}{l_1 - l_a} + \frac{1}{l_3 - l_c}\right) \quad \text{.....(2)}$$

$$l_3 = l_{eq} - l_1$$

Solving (1) and (2), $l_c =$ m (or) m

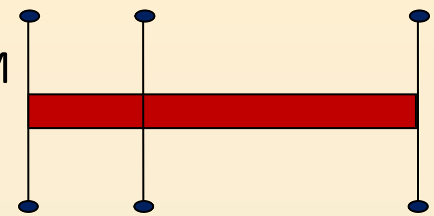
Answer1: la > l1 and lc <l2 $l_a > l_1 \; and \; l_c < l_2$ Hence single node system

Answer2: lc < l1 and lc <l2 $l_a < l_1 \; and \; l_c < l_2$ Hence two nodes system

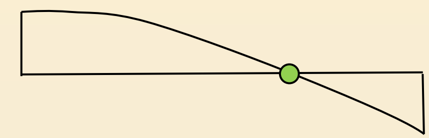
$$J = \frac{\pi}{32} d^4 \qquad \omega_n = \sqrt{\frac{CJ}{I_a l_a}}$$

DYNAMICS OF MACHINERY

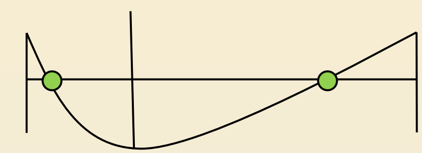
NATURAL FREQUENCY OF THREE ROTOR SYSTEM



Given: Three rotors A, B, C; Shaft diameter d = 0.07 m;
Modulus of rigidity C = 84x 10 ^{9} N/m^2
Distance between A and B = 1.5 m;
Distance between B and C = 1.0 m



	A	B	C	
Moment of Inertia	0.3	0.6	0.18	kgm^2



To find (a) Frequency of torsional vibrations; (b) position of nodes;
(c) Amplitudes of vibrations

I_A = 0.3 kgm^2; I_B =0.6 kgm^2; I_C = 0.18 kgm^2; d = 0.07m; C = 8.4E10 N/m^2 ; L_1 = 1.5 m; L_2 = 1.0 m;

In a three rotor system, $f_{nA} = f_{nB} = f_{nC}$; $I_a l_a = I_b l_b = I_c l_c$

$$l_{eq} = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4$$

$$l_3 = l_{eq} - l_1$$

$$l_a = \frac{I_c}{I_a} l_c \qquad \text{.....(1)}$$

$$\frac{1}{I_c l_c} = \frac{1}{I_b}\left(\frac{1}{l_1 - l_a} + \frac{1}{l_3 - l_c}\right) \qquad \text{.....(2)}$$

Solving (1) and (2), $l_c =$ m (or) m

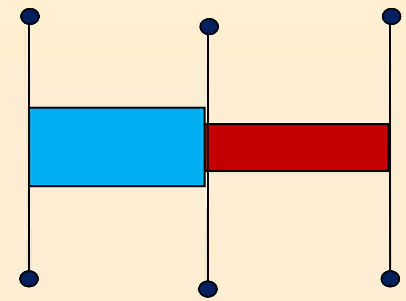
Answer1: $l_a > l_1 \; and \; l_c < l_2$ Hence single node system

Answer2: $l_a < l_1 \; and \; l_c < l_2$ Hence two nodes system

$$J = \frac{\pi}{32} d^4 \qquad \omega_n = \sqrt{\frac{CJ}{I_a l_a}}$$

DYNAMICS OF MACHINERY

NATURAL FREQUENCY OF THREE ROTOR SYSTEM



Given: Three rotors A, B, C;
Modulus of rigidity C = 80x 10 ^{9} N/m^2
Distance between A and B = 0.05 m dia, 2 m long;
Distance between B and C = 0.025 m dia, 2 m long

	Engine A	Flywheel B	Propeller C	
Moment of Inertia	800	320	20	kgm^2

To find (a) Natural Frequency of torsional vibrations; (b) position of nodes;

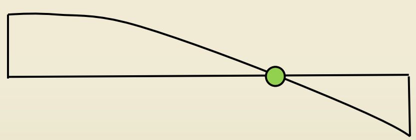
I_A = 800 kgm^2; I_B =320 kgm^2; I_C = 20 kgm^2; d1 = 0.05 m;d2= 0.025 m; C = 8.0E10 N/m^2 ;
L_1 = 2.0 m; L_2 = 2.0 m; In a three rotor system, $f_{nA} = f_{nB} = f_{nC}$; $I_a l_a = I_b l_b = I_c l_c$

$$l_a = \frac{I_c}{I_a} l_c \quad .....(1)$$

$$l_{eq} = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4$$

$$\frac{1}{I_c l_c} = \frac{1}{I_b}\left(\frac{1}{l_1 - l_a} + \frac{1}{l_3 - l_c}\right) \quad .....(2)$$

$$l_3 = l_{eq} - l_1$$



Solving (1) and (2), $l_c =$ m (or) m

Answer1: $l_a > l_1 \; and \; l_c < l_2$ Hence single node system

Answer2: $l_a < l_1 \; and \; l_c < l_2$ Hence two nodes system

$$J = \frac{\pi}{32} d^4$$

$$\omega_n = \sqrt{\frac{CJ}{I_a l_a}}$$

END OF UNIT 3