UNIT III
CASH FLOW

by

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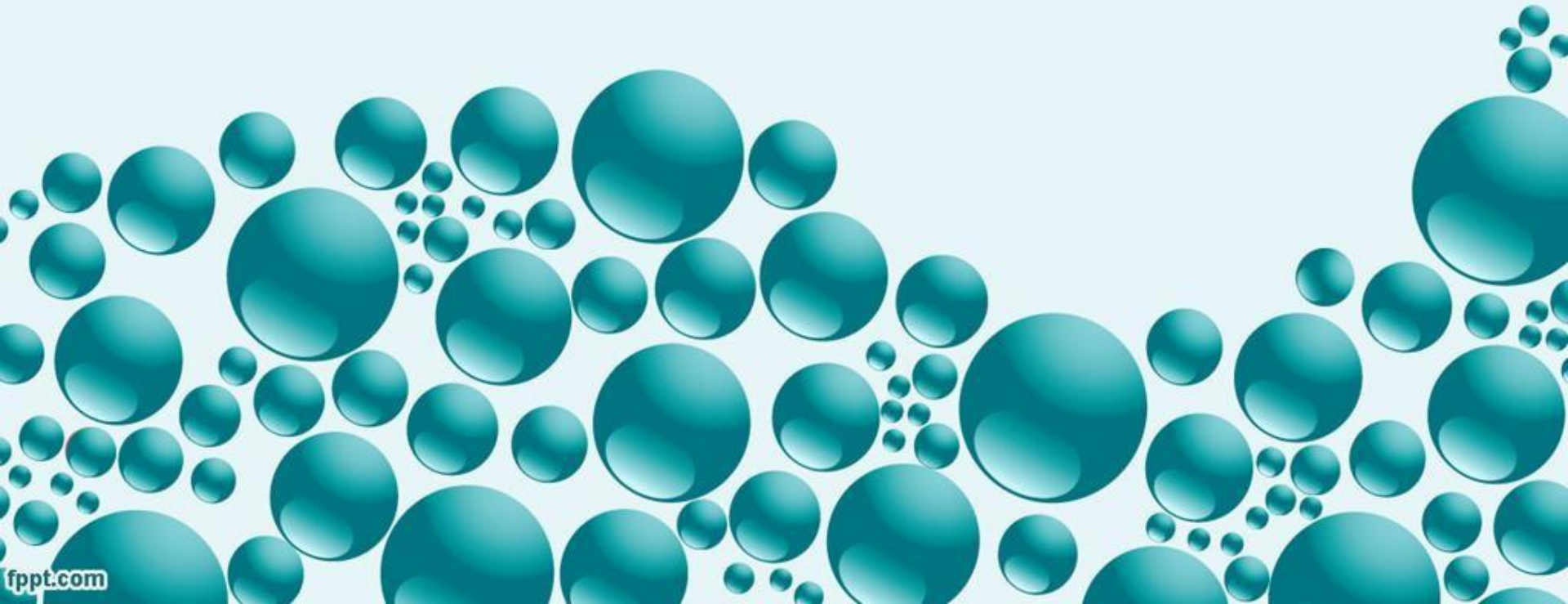

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PRESENT WORTH METHOD OF COMPARISON

- Among the different alternative the best alternative is selected based on the value of the present worth of different alternatives.
- In revenue/profit dominated cash flow diagram :

  (a) all inflows to the organization such as profit, revenue, salvage value is (+)

  (b) The cost (Outflows) will be assigned with (-)

PRESENT WORTH METHOD OF COMPARISON

- In the cost dominated cash flow diagram:

  (a) The cost (Outflows) will be assigned with (+)

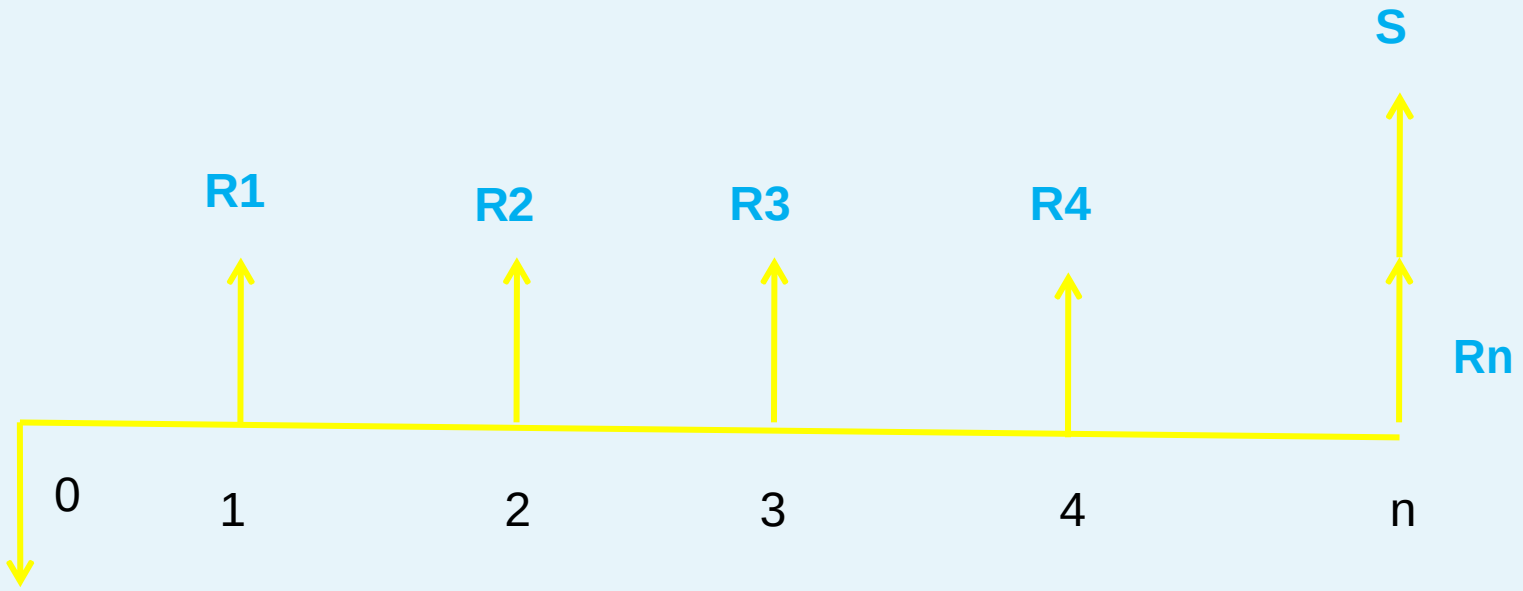
  (b) All inflows to the organization such as profit, revenue, salvage value is (-)

DECISION ON PRESENT WORTH METHOD OF COMPARISON

- To select an alternative with minimum cost then the alternative with least present worth amount will be selected.

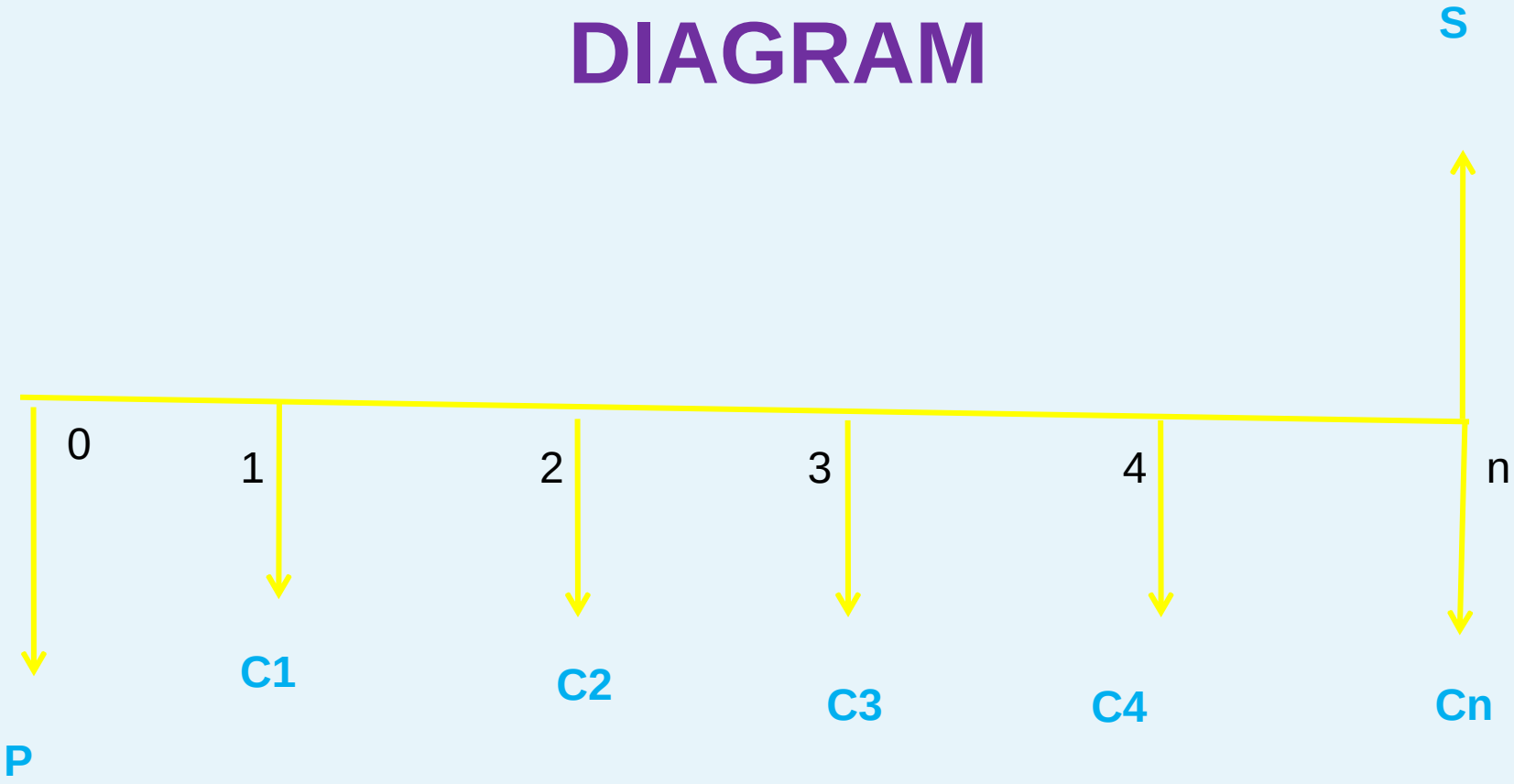
- To select an alternative with maximum profit then the alternative with maximum present worth amount will be selected

REVENUE DOMINATED CASH FLOW DIAGRAM



$$PW(i) = -P + R1\left[\frac{1}{(1+i)^1}\right] + R2\left[\frac{1}{(1+i)^2}\right] + R3\left[\frac{1}{(1+i)^3}\right] + \cdots + Rn\left[\frac{1}{(1+i)^n}\right] + S\left[\frac{1}{(1+i)^n}\right]$$

COST DOMINATED CASH FLOW DIAGRAM



PW (i) = $$P + C1\left[\frac{1}{(1+i)^1}\right] + C2\left[\frac{1}{(1+i)^2}\right] + C3\left[\frac{1}{(1+i)^3}\right] + \cdots Cn\left[\frac{1}{(1+i)^n}\right] - S\left[\frac{1}{(1+i)^n}\right]$$

PROBLEM

- A project involves an initial cost of Rs 30,00,000 and with the following transactions for the next five years. The salvage value at the end of the life of the project is Rs 2,00,000. Draw a cash flow diagram for the project and find its present worth assuming i = 15% compounded annually

End of year	Maintenance and Operating expense (Rs)	Revenue (Rs)
1	2,00,000	9,00,000
2	2,50,000	10,00,000
3	3,00,000	12,00,000
4	3,00,000	13,00,000
5	4,00,000	12,00,000

SOLUTION

Given: Initial outlaay = Rs.30,00,000
Salvage valuue = Rs.2,00,000
Interest rate (i) = 15%, compounded annually
Liffe = 5 years

To find: The present - worth of the project = PW(i)

The cost dominated cash flow diagram of project is shown in Fig.3.3.

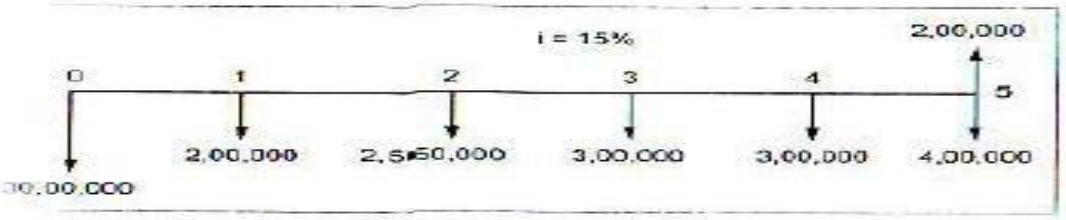


Fig. 3.3. Cash flow diagram for the project

☺ *Solution:*

PW (15%)

= 30,00,000 + 2,00,0000 (P/F, 15%, 1)
+ 2,50,000 (P/F, 15%, 2) + 3,00,000 (P/F, 15%, 3)
+ 3,00,000 (P/F, 15%, 4) + 4,00,000 (P/F, 15%, 5)
– 2,00,000 (P/F, 15%, 5)

= 30,00,000 + 2,00,0000 × 0.8696 + 2,50,000
× 0.7561 + 3,00,000 × 0.6575 + 3,00,000 × 0.5718
+ 4,00,000 × 0.4972 – 2,00,000 × 0.4972

= 30,00,000 + 1,73,9220 + 1,89,025 + 1,97,250
+ 1,71,540 + 1,98,880 – 99,440

= Rs.38,31,175

For the revenue dominated cash flow diagram for the project is shown in Fig.3.4.

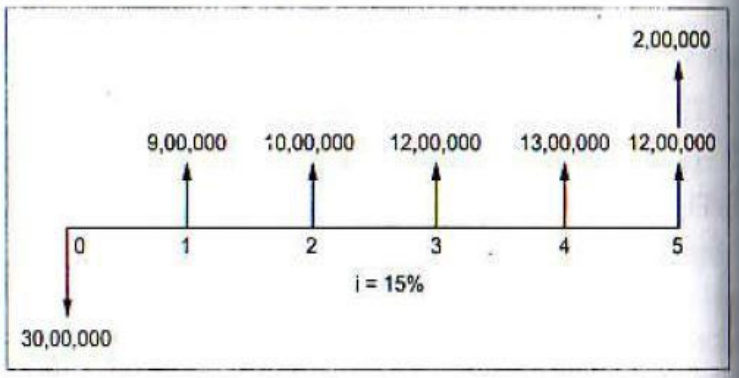


Fig. 3.4. Cash flow diagram for the project

☺ ***Solution:***

PW (15%)

$= -30,00,000 + 9,00,000\ (P/F, 15\%, 1)$
$+ 10,00,000\ (P/F, 15\%, 2) + 12,00,000\ (P/F, 15\%, 3)$
$+ 13,00,000\ (P/F, 15\%, 4) + 12,00,000\ (P/F, 15\%, 5)$
$+ 2,00,000\ (P/F, 15\%, 5)$

$= -30,00,000 + 9,00,000\ (0.8696) + 10,00,000$
$(0.7561) + 12,00,000\ (0.6575) + 13,00,000\ (0.5718)$
$+ 12,00,000\ (0.4972) + 2,00,000\ (0.4972)$

$= -30,00,000 + 7,82,640 + 7,56,100 + 7,89,000$
$+ 7,43,340 + 5,96,640 + 99,440$

= Rs.7,67,160

Result: Expression of cost dominated cash flow diagram = Rs.38,31,175

Expression of revenue dominated cash flow diagram = Rs.7,67,160

Example 3.2 *Find the present worth of the following cash flow series. Assume i = 15% compounded annually.*

End of year	0	1	2	3	4	5
Cash flow (Rs.)	–10,000	30,000	30,000	30,000	30,000	30,000

Given: Initial outlay = 10,000
Annual revenue = 30,000
Interest rate = 15%, compounded annually
Life = 5 years

To find: Present worth of 15%

The cash flow diagram for series is shown in Fig.3.5.

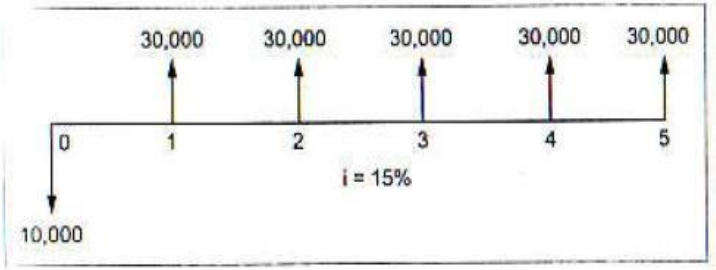


Fig. 3.5. Cash flow diagram for the series

☺ ***Solution:***

PW (15%) $= -10,000 + 30,000 \times (P/A, 15\%, 5)$
$= -10,000 + 30,000 \times 3.3522$
$= -10,000 + 1,00,566$
= Rs.90,566

Result: The present worth of the given cash flow series is Rs.90,566 and it is feasible.

Example 3.3 *Consider the following cash flow series over a 20 year period. Assuming the interest rate as 18% compounded annually, compute the present worth of the series; give your comment.*

End of year	Cash flow (Rs.)
0	– 50,00,000
1	6,00,000
2	6,00,000
⋮	⋮
20	6,00,000

Given: Initial outlay = – 50,00,000

Annual equivalent revenue = 6,00,000

Interest rate (i) = 18%, compounded annually

Life = 20 years

To find: The present worth PW (18%)

The cash flow diagram for the cash flow series is shown in Fig.3.6.

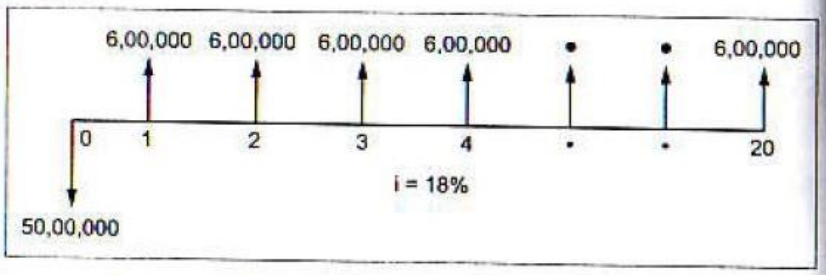


Fig. 3.6. Cash flow diagram for the cash flow series

☺ ***Solution:***

PW (18%) = –50,00,000 + 6,00,000 (P/A, 18%, 20)

= –50,00,000 + 6,00,000 × 5.3527

= –50,00,000 + 32,11,620

= Rs.–17,88,380

Result: The present worth of the cash flow series is Rs. –17,88,380. Therefore, it is not feasible.

Example 3.4 *The cost of creating an oil well is Rs 1,50,00,000. The annual equivalent yield from the oil well is Rs.30,00,000. The salvage value after its useful life of 10 years is Rs.2,00,000. Assuming an interest rate of 18%, compounded annually, find out whether the erection of the oil well is financially feasible, based on the present worth method.*

Given: Initial cost = Rs.1,50,00,000

Annual equivalent yield = Rs.30,00,000

Salvage value = Rs.2,00,000

Life = 10 years

Interest rate = 18%, compounded annually.

To find: The present worth of the oil well PW (18%)

The cash flow diagram for the oil well is shown in Fig.3.7.

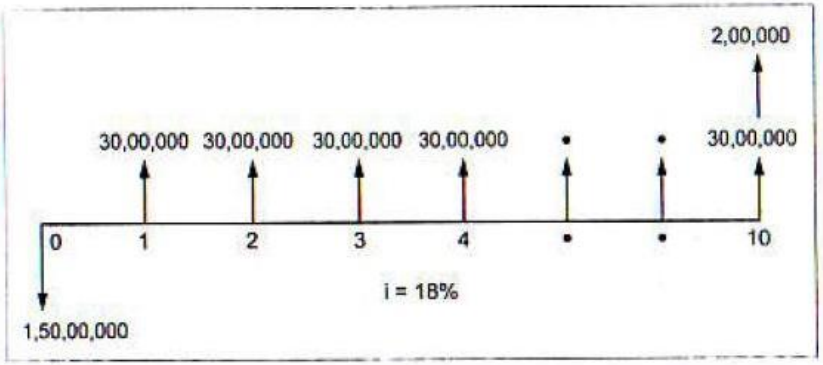


Fig. 3.7. Cash flow diagram for the oil well

☺ ***Solution:***

$$PW(18\%) = -1,50,00,000 + 30,00,000\ (P/A, 18\%, 10) + 2,00,000\ (P/F, 18\%, 10)$$
$$= -1,50,00,000 + 30,00,000 \times 4.4941 + 2,00,000 \times 0.1911$$
$$= -1,50,00,000 + 1,34,82,300 + 38,220$$
$$= \text{Rs. } -14,79,480$$

Result: The present worth of the oil well is Rs. –14,79,480. As it is in negative sign, the oil well is not feasible.

Example 3.5 *The Details of the feasibility report of a project are as shown below. Check the feasibility of the project based on present worth method, using i = 20%.*

Initial outlay = *Rs. 50,00,000*

Life of the project = *20 years*

Annual equivalent revenue = *Rs. 15,00,000*

Salvage value at the end of project life = *Rs. 5,00,000*

Given: Initial outlay = Rs. 50,00,000

Life of the project = 20 years

Annual revenue = Rs. 15,00,000

Salvage value = Rs. 5,00,000

To find: The present worth of the project PW (20%)

The cash flow diagram of the project is shown in Fig.3.8.

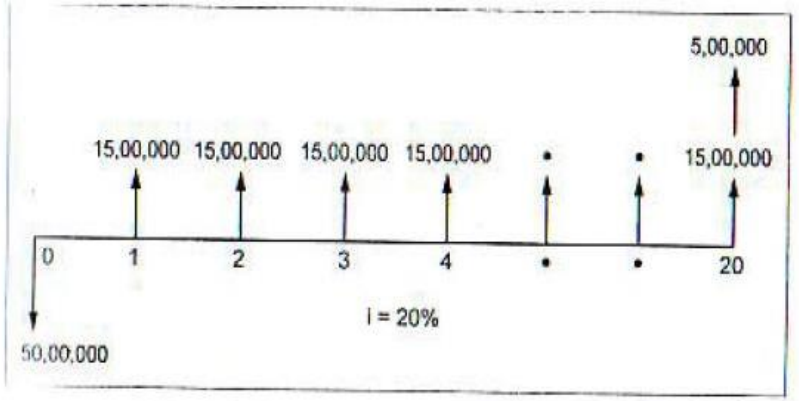


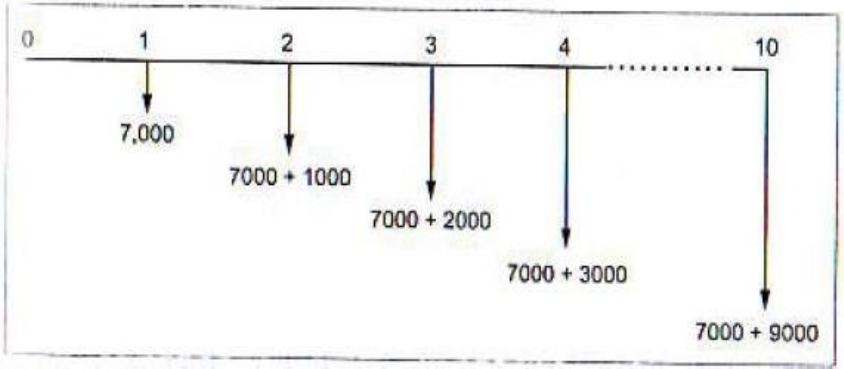
Fig. 3.8. Cash flow diagram of the project

☺ ***Solution:***

$$PW\ (20\%) = -50,00,000 + 15,00,000\ (P/A, 20\%, 20) + 5,00,000\ (P/F, 20\%, 20)$$
$$= -50,00,000 + 15,00,000 \times 4.8694 + 5,00,000 \times 0.0261$$
$$= -50,00,000 + 73,04,400 + 13,050$$
$$= \text{Rs. } 23,17,450$$

Result: The present worth of the project is more than 0. So, the project is feasible.

Example 3.6 *Consider the following cash flow diagram. Find the present worth using an interest rate of 15%, compounded annually.*



Given: Expenditure during the first year = Rs.7000
Annual increase in expenditure, G = Rs.1000
Life (n) = 10 years
Interest rate (i) = 15%, compounded annually.

To find: The present worth of the cash flow diagram

☺ ***Solution:***

PW (15%) = 7000 + [1000 (A/G, 15%, 10)]
× (P/A, 15%, 10)
= 7000 + [1000 (3.3832)] × 5.0188
= 7000 + 3383.2 × 5.0188
= 7000 + 16979.60
= Rs.23,979.60

Result: The present worth of the cash flow diagram is Rs.23,979.60.

Example 3.7 *An automobile company recently advertised its car for a down payment of Rs.1,50,000. Alternatively, the car can be taken home by customers without making any payment but they have to pay yearly amount of Rs.25,000 for 15 years at an interest rate of 18%, compounded annually. You are asked to advise the best alternative for the customers based on the present worth method of comparison.*

Given:

There are two alternatives available for the company

1. Down payment = Rs.1,50,000
2. Equal installment = Rs.25,000
   Life = 15 years
   Interest rate (i) = 18%, compounded annually.

To find: The present worth of the alternative 2 PW(18%)

The cash flow diagram of the second alternative is shown in Fig.3.9.

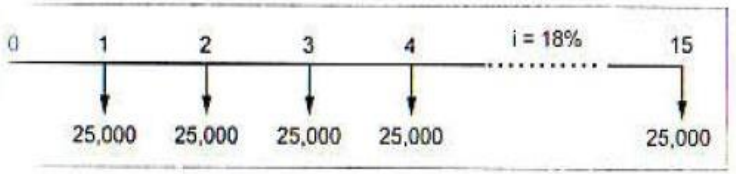


Fig. 3.9. Cash flow diagram for alternative 2

☺ ***Solution:*** $PW_2(18\%)$ = 25,000 (P/A, 18%, 15)
= 25,000 (5.0916)
= Rs.1,27,290

Result: While comparing the both the alternative, the second alternative is less than the first alternative. Therefore, the company can go for the second alternative.

Example 3.8 *The cash flows of two project proposals are as given below. Each of the project has an expected life of 10 years. Select the best project based on present worth method of comparison using an interest rate of 18%, compounded annually.*

	Initial outlay (Rs.)	Annual equivalent revenue (Rs.)	Salvage value after 10 years (Rs.)
Project 1	– 7,50,000	2,00,000	50,000
Project 2	– 9,50,000	2,25,000	1,00,000

Project 1:

Given: Initial outlay = –7,50,000

Annual revenue = 2,00,000

Salvage value = 50,000

Interest rate (i) = 18%, compounded annually.

Life = 10 years

To find: The present worth for the project 1 $PW_1(18\%)$

The cash flow diagram for the project 1 is shown in Fig.3.10.

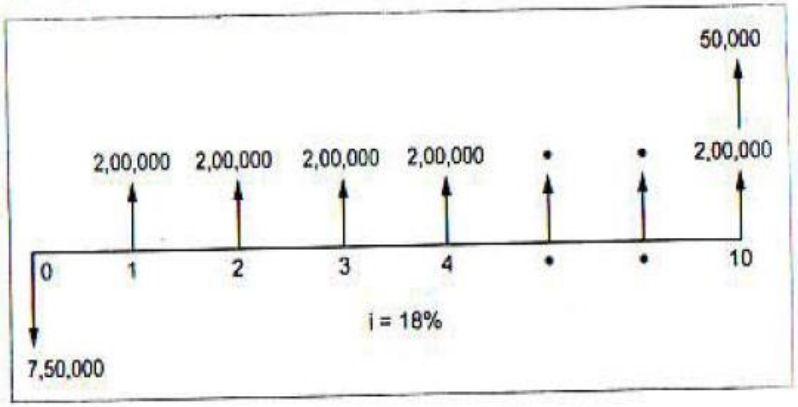


Fig. 3.10. Cash flow diagram for project 1

☺ ***Solution:***

$PW_1(18\%)$ = –7,50,000 + 2,00,000 (P/A, 18%, 10)
+ 50,000 (P/F, 18%, 10)

= –7,50,000 + 2,00,000 × 4.4941 + 50,000 × 0.1911

= –7,50,000 + 8,98,820 + 9555

= Rs.1,58,375

Project 2:

Given: Initial outlay = – 9,50,000

Annual revenue = 2,25,000

Salvage value = 1,00,000

Interest rate (i) = 18%, compounded annually.

Life = 10 years

To find: The present worth for the project 2 $PW_2(18\%)$

The cash flow diagram for the project 2 is shown in Fig.3.11.

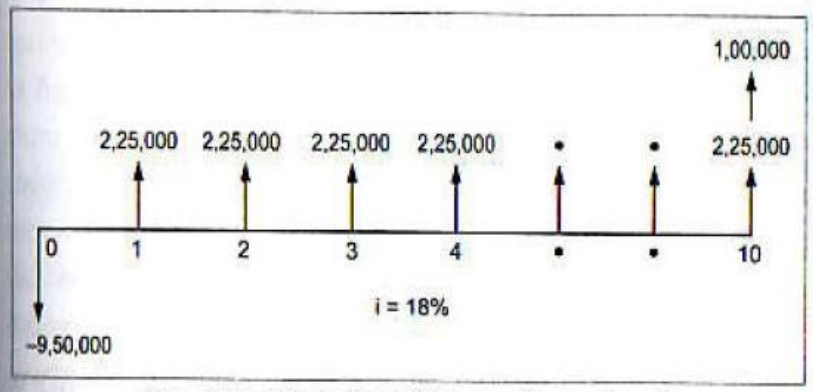


Fig. 3.11. Cash flow diagram for project 2

☺ ***Solution:***

$PW_2(18\%)$ = –9,50,000 + 2,25,000 (P/A, 18%, 10)
+ 1,00,000 (P/F, 18%, 10)

= –9,50,000 + 2,25,000 × 4.4941 + 1,00,000 × 0.1911

= –9,50,000 + 10,11,172.5 + 19,110

= Rs.80,282.5

Result: While comparing the present worth of the project 1 & project 2, the project one is more than the project 2. So the project one is the best alternative.

Example 3.9 *A company has two alternative for satisfying its daily travel requirements of its employees for the next five years.*

Alternative 1: Renting a vehicle at a cost of Rs.10,00,000 per year.

Alternative 2: Buying a vehicle for Rs.5,00,000 with an operating and maintenance cost of Rs.3,50,000 per year. The salvage value of the vehicle after five years is Rs.1,00,000.

Select the best alternative based on the present worth method of comparison using the interest rate of 20%, compounded annually.

Given:

Alternative 1:

Renting a vehicle at a cost of Rs.10,00,000

Alternative 2:

Initial outlay = Rs.5,00,000

Operating & maintenance cost = Rs.3,50,000

Salvage value = Rs.1,00,000

Life = 5 years

Interest rate = 20%, compounded annually.

To find: The present worth of the alternative 2 PW_2 (20%)

The cash flow diagram for alternative 2 is shown in Fig.3.12.

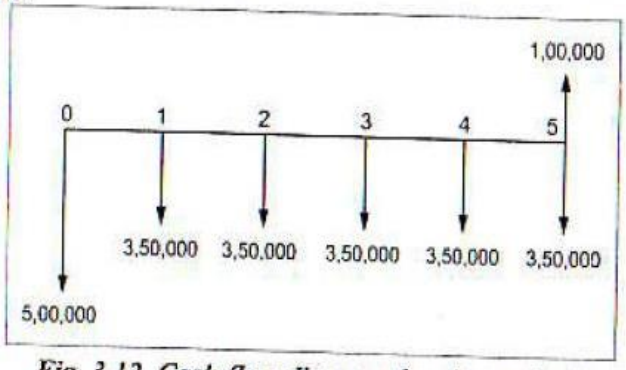


Fig. 3.12. Cash flow diagram for alternative 2

☺ ***Solution:***

$PW_2(20\%) = 5,00,000 + 3,50,000\ (P/A, 20\%, 5)$
$- 1,00,000\ (P/F, 20\%, 5)$

$= 5,00,000 + 3,50,000 \times 2.9906 - 1,00,000 \times 0.4019$

$= 5,00,000 + 10,46,710 - 40,190$

$=$ Rs.15,06,520

Result: Renting the vehicle is the best alternative, because the present worth of the alternative two is more expensive.

Example 3.10 *A working woman is planning for her retired life. She has 20 more years of service. She would like to have an annual equivalent amount of Rs.3.00,000 starting from the end of the first year of her retirement. Find the single amount that should be deposited now so that she receives the above mentioned annual equivalent amount at the end of every year for 20 years after her retirement. Assume i = 15%, compounded annually.*

Given: Annual return = Rs.3,00,000

Interest rate (i) = 15%, compounded annually.

Life = 20 years

To find: Present worth PW (15%)

The cash flow diagram for the plan is shown in Fig.3.13.

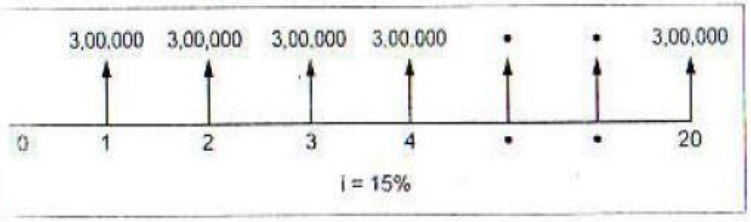


Fig. 3.13. Cash flow diagram for the plan

☺ ***Solution:*** PW(15%) = 3,00,000 (P/A, 15%, 20)

= 3,00,000 (6.2593)

= Rs.18,77,790

Result: The single amount that should deposited now is Rs.18,77,790.

Example 3.11 *An engineer is considering two types of pressure sensors for a low pressure steam line. The costs are shown below which should be selected based on a present worth comparison at an interest rate of 16% per year?*

[Anna university, April/may 2008]

	Type X	Type Y
First cost	Rs.76,500	Rs.1,29,000
Maintenance cost/year	Rs.12,000	Rs.9,000
Salvage value	0	20,000
Life, years	2	4

Type X:

Given: First cost = Rs.76,500

Maintenance cost/year = Rs.12,000

Salvage value = 0

Life = 2 years

Interest rate = 16%

To find: PW_X (15%)

The cash flow diagram of Type X is shown in Fig.3.14.

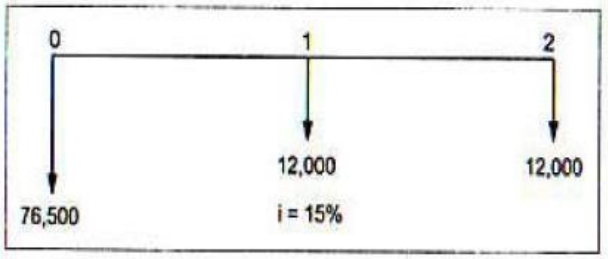


Fig. 3.14. Cash flow diagram for the plan

☺ ***Solution:***

$PW_X(15\%) = 76,000 + 12,000\ (P/A, 16\%, 2)$

$= 76,000 + 12,000 \times 1.6052$

$= 76,000 + 19,262.40$

$=$ Rs. 95,262.40

Type Y:

Given: First cost = Rs.1,29,000

Maintenance cost/year = Rs.9,000

Salvage value = 20,000

Life = 4 years

Interest rate = 16%

To find: PW_Y (16%)

The cash flow diagram of Type Y is shown in Fig.3.15.

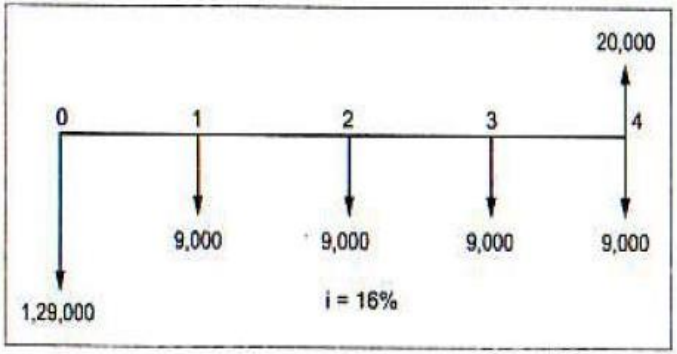


Fig. 3.15. Cash flow diagram for the plan

☺ ***Solution:***

$$PW_Y(16\%) = 1,29,000 + 9000\ (P/A, 16\%, 4) - 20,000\ (P/F, 16\%, 4)$$

$$= 1,29,000 + 9000 \times 2.7982 - 20,000 \times 0.5523$$

$$= 1,29,000 + 25183.8 - 11,046$$

$$= \text{Rs. } 1,43,137.8$$

Result: The present worth of this option is Rs.1,43,137.8 which is more than the first option of complete down payment of Rs.95,262.40. Hence, the engineer has to select the type X of pressure sensors for a low pressure steam line.

3.1.4. Exercise Problems

1. Devi industry is planning to expand its production operation. It has identified three different technologies for meeting the goal. The initial outlay and annual revenues with respect to each of the technologies are summarized below. Suggest the best technology which is to be implemented based on the present worth method of comparison assuming 20% interest rate, compounded annually.

	Initial outlay (Rs.)	**Annual revenue (Rs.)**	**Life**
Technology 1	12,00,000	4,00,000	10 years
Technology 2	20,00,000	6,00,000	10 years
Technology 3	18,00,000	5,00,000	10 years

FUTURE WORTH METHOD

FUTURE WORTH METHOD

- Future worth method is used particularly in an investment situation where we need to compute the equivalent worth of the project at the end of its investment period
- For Eg : Building a nuclear power plant, where it is time consuming. In such situation it is more common to measure the worth of the investment at the time of commercialization

THE ACCEPT AND REJECT RULE

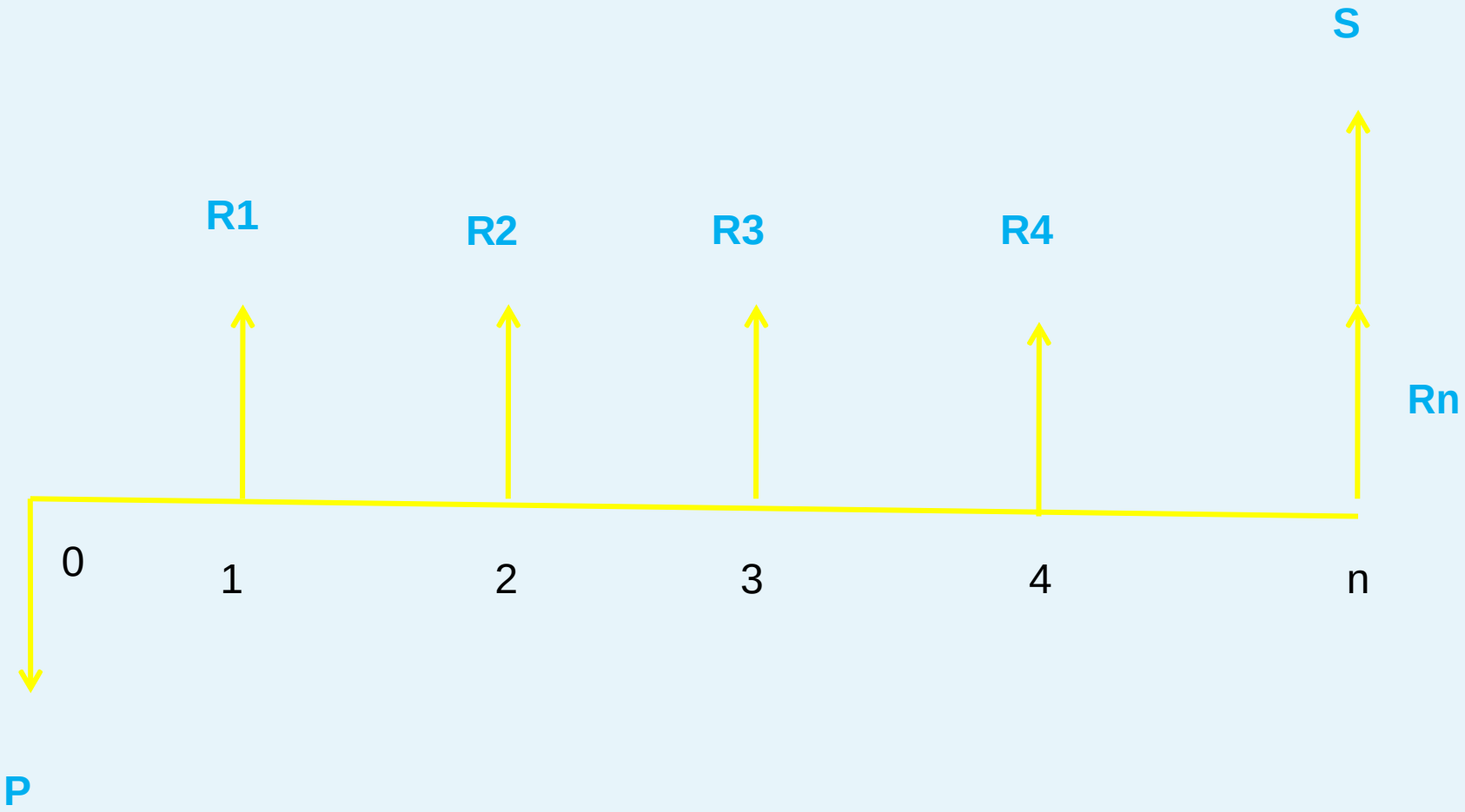
- The accept and reject decision rule for a single project evaluation is as follows:

  If FW(i) > 0, accept the investment

  If FW(i)= 0, Remain indifferent to the investment

  If FW(i) < 0, Reject the investment

REVENUE DOMINATED CASH FLOW DIAGRAM



SIGN CONVENTION

- In revenue/profit dominated cash flow diagram :

  (a) all inflows to the organization such as profit, revenue, salvage value is (+).

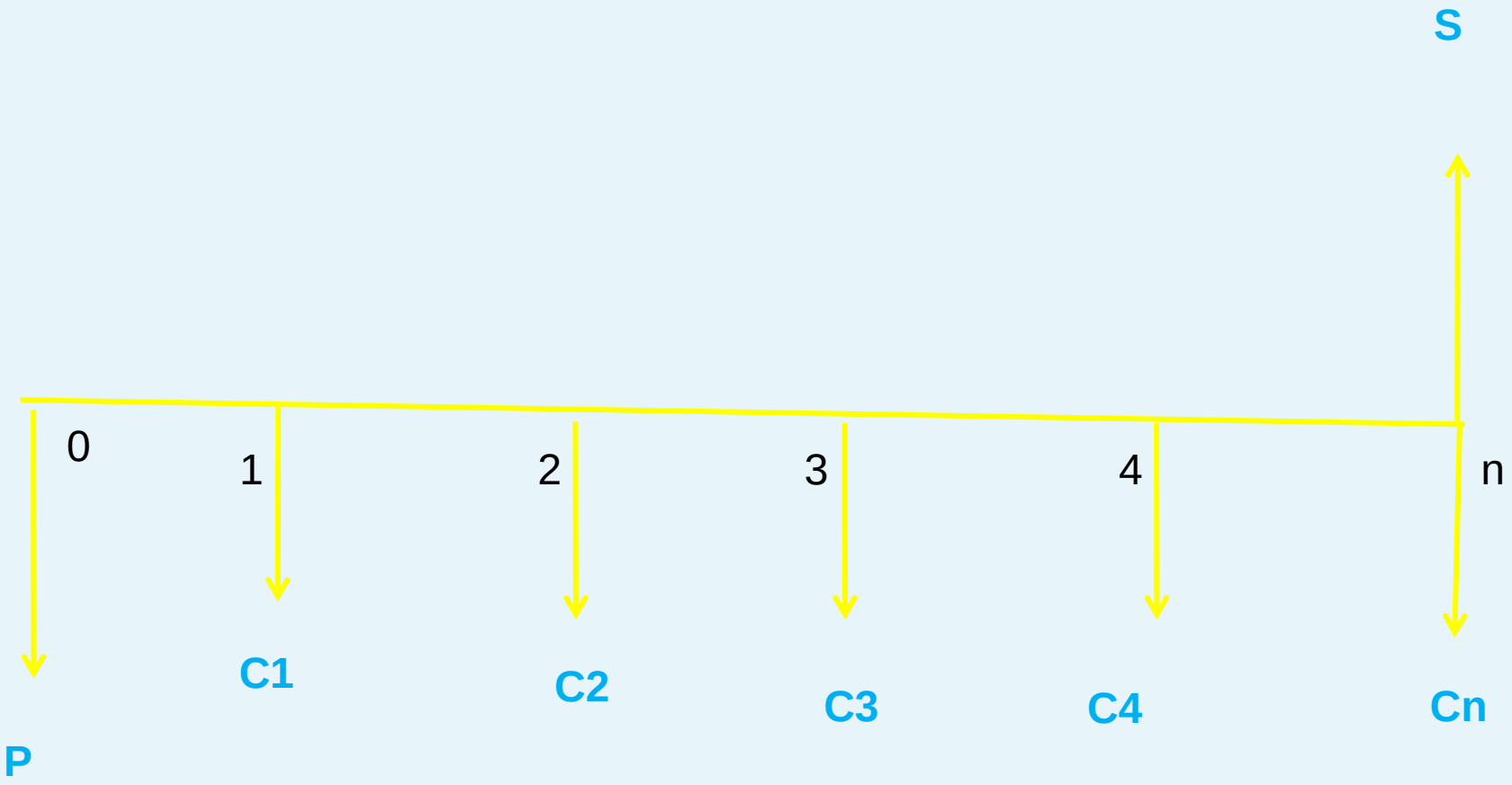
  (b) The cost (Outflows) will be assigned with (-).

- The formula for computing the future worth of the above diagram

  FW(i) = $-P(1+i)^n + R1(1+i)^{n-1} + R2(1+i)^{n-2} + R3(1+i)^{n-3} + R4(1+i)^{n-4} \ldots + Rn + S$

- The alternative with maximum future worth amount should be selected as the best alternative.

COST DOMINATED CASH FLOW DIAGRAM



SIGN CONVENTION

- In the cost dominated cash flow diagram:

  (a) The cost (Outflows) will be assigned with (+)

  (b) All inflows to the organization such as profit, revenue, salvage value is (-)

- The formula for computing the future worth of the diagram is

- $$FW(i) = P(1+i)^n + C1(1+i)^{n-1} + C2(1+i)^{n-2} + C3(1+i)^{n-3} + C4(1+i)^{n-4} \ldots + Cn - S$$

- The alternative with minimum future worth amount should be selected as the best alternative.

PROCEDUE TO CALCULTE ANNAUL EQUIVALENT REVENUE

- Step 1 : Calculate the present worth value of the given alternatives

- Step 2 : Calculate the annual equivalent revenue using the present worth value

ANNUAL EQUIVALENT METHOD

- REVENUE DOMINATED CASH FLOW DIAGRAM

PW(i) $-P+R1\left[\frac{1}{(1+i)^1}\right]+R2\left[\frac{1}{(1+i)^2}\right]+R3\left[\frac{1}{(1+i)^3}\right]+\cdots+Rn\left[\frac{1}{(1+i)^n}\right]+S\left[\frac{1}{(1+i)^n}\right]$

- COST DOMINATED CASH FLOW DIAGRAM

PW(i) = $P+C1\left[\frac{1}{(1+i)^1}\right]+C2\left[\frac{1}{(1+i)^2}\right]+C3\left[\frac{1}{(1+i)^3}\right]+\cdots Cn\left[\frac{1}{(1+i)^n}\right]-S\left[\frac{1}{(1+i)^n}\right]$

TO CALCULATE ANNUAL EQUIVALENT REVENUE

- Using the formula

$$A = PW(i)\frac{i(1+i)^n}{(1+i)^n - 1}$$

- Using the Tables :

$$A = PW(i)\ (A/P\ ,\ i,\ n)$$

DECISION ON ANNAUL EQUIVALENT METHOD

- For various alternative if it is a revenue dominated method choose the alternative with maximum annual equivalent revenue.

- For various alternatives if it is a cost dominated method choose the alternative with minimum annual equivalent revenue.

RATE OF RETURN METHOD

- The rate of return of a cash flow pattern is the rate of interest at which the present worth of the cash flow reduces to zero.
- The rate of return for each alternative is computed.
- The alternative which has the highest rate of return is selected as the best alternative.

PROCEDURE

- Step 1 : To find the present worth of the given alternative using some i value

PW(i)= $$-P+R1\left[\frac{1}{(1+i)^1}\right]+R2\left[\frac{1}{(1+i)^2}\right]+R3\left[\frac{1}{(1+i)^3}\right]+\cdots+Rn\left[\frac{1}{(1+i)^n}\right]+S\left[\frac{1}{(1+i)^n}\right]$$

- Step 2 : The above procedure is repeated till the present worth value changes from positive to negative.
- Step 3 : Then the value of i is obtained using the method of interpolation

PRESENT WORTH FUNCTION GRAPH