The Method of Joints

- For design analysis of a truss, we need to obtain the force in each of the members
- Considering the FBD, the forces in the members are internal forces and could not be obtained from an equilibrium analysis
- Considering the equilibrium of a joint of the truss, a member force becomes an external force on the joint's FBD and equations of equilibrium can be applied
- This forms the basis for the method of joints

The Method of Joints

- Truss members are all straight two force members lying in the same plane
- The force system acting at each joint is coplanar and concurren
- Rotational or moment equilibrium is automatically satisfied at the pin
- $\sum F_x = 0$ and $\sum F_y = 0$ must be satisfied for equilibrium

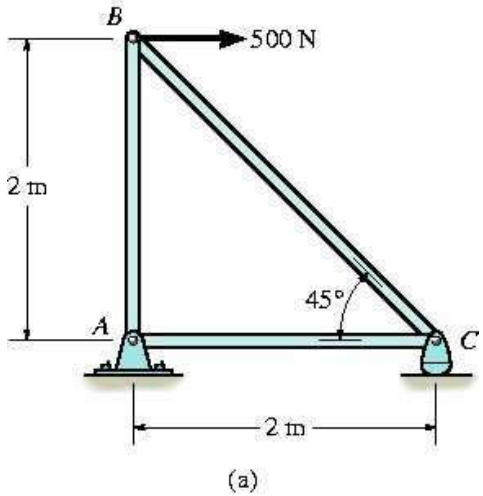
The Method of Joints

Method of Joints

- Draw FBD
- Line of action of each member force acting on the joint is specified from the geometry of the truss since the force in a member passes along the axis of the member

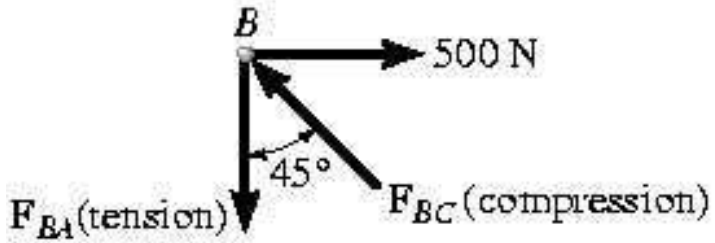
Example

- Consider pin at joint B
- Three forces: 500N force and forces exerted by members BA and BC

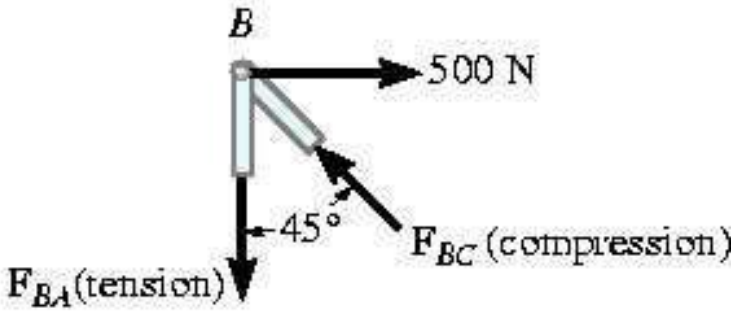


6.2 The Method of Joints

- $\mathbf{F}_{BA}$ is "pulling" on the pin, meaning the member BA is in tension
- $\mathbf{F}_{BC}$ is "pushing" on the pin, meaning the member BC is in compression
- The pushing and pulling indicates the effect of the member being either in tension or compression



(b)



(c)

6.2 The Method of Joints

Determining the Correct Sense of the Unknown Member

- Always assume the unknown member forces acting on the joint's FBD to be in tension

  -The numerical solution of the equilibrium will yield positive scalars for members in tension and negative scalars for members in compression

  -Use the correct magnitude and sense of the unknown member on subsequent FBD

6.2 The Method of Joints

Determining the Correct Sense of the Unknown Member

- The correct sense of a direction of an unknown force can be determined by inspection

  $-\mathbf{F}_{BC}$ must push on the pin (compression) since its horizontal component must balance the 500N force

  $-\mathbf{F}_{BA}$ is a tensile force since it balances the vertical component of $\mathbf{F}_{BC}$

6.2 The Method of Joints

Determining the Correct Sense of the Unknown Member

- The correct sense of a direction of an unknown force can be determined by inspection

  -In more complicated problems, the sense of the member can be assumed

  -A positive answer indicates that the assumed sense is correct and a negative answer indicates that the assumed sense must be reversed

6.2 The Method of Joints

Procedure for Analysis

- Draw the FBD of a joint having at least one known force and at most two unknown forces
- If this joint is at one of the supports, determine the external reactions at the truss support
- Use one of two methods for determining the correct sense of the member
- Orient the x and y axes so that the forces on the FBD can be easily resolved into x and y components

6.2 The Method of Joints

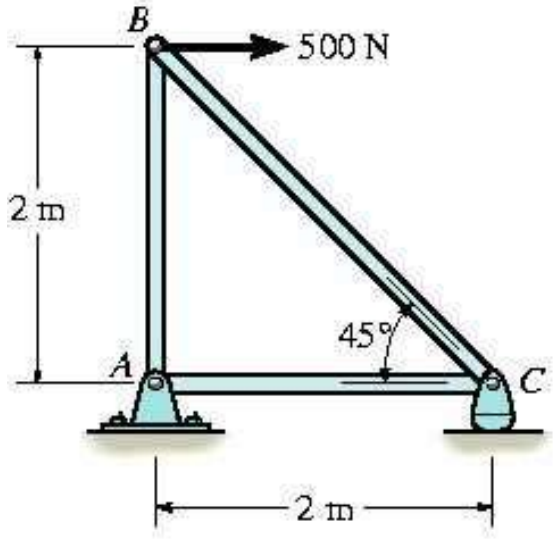
Procedure for Analysis

- Apply $\Sigma F_x = 0$ and $\Sigma F_y = 0$
- Solve for unknown members forces and verify their correct sense
- Continue to analyze each of the other joints
- Once the force in a member is found from the analysis of the joint at one of its end, the result is used to analyze the forces acting on the other end

6.2 The Method of Joints

Example 6.1

Determine the force in each member of the truss and indicate whether the members are in tension or compression.



(a)

6.2 The Method of Joints

Solution

- Two unknown member forces at joint B
- One unknown reaction force at joint C
- Two unknown member forces and two unknown reaction forces at point A

6.2 The Method of Joints

Solution

Joint B

$$+\rightarrow \sum F_x = 0;$$

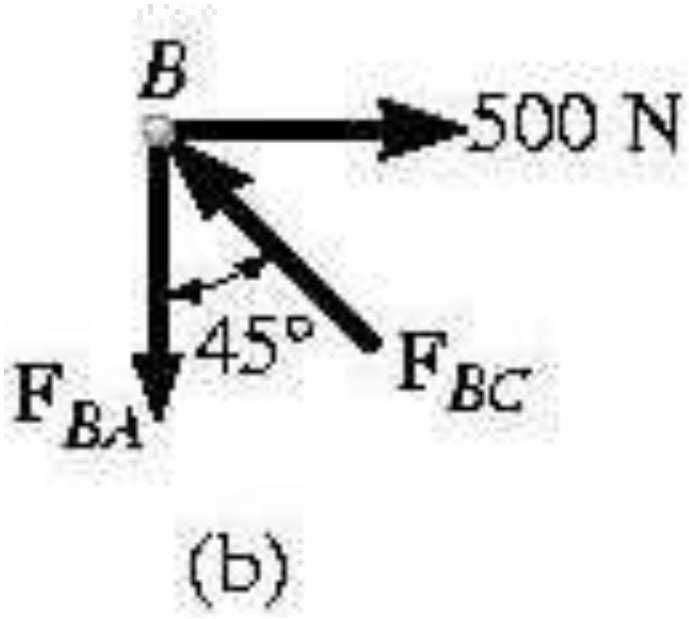
$$500N - F_{BC}\sin 45^{\circ}N = 0$$

$$F_{BC} = 707.1N(C)$$

$$+\uparrow \sum F_y = 0;$$

$$F_{BC}\cos 45^{\circ}N - F_{BA} = 0$$

$$F_{BA} = 500N(T)$$



(b)

6.2 The Method of Joints

Solution

Joint C

$$+\rightarrow \sum F_x = 0;$$

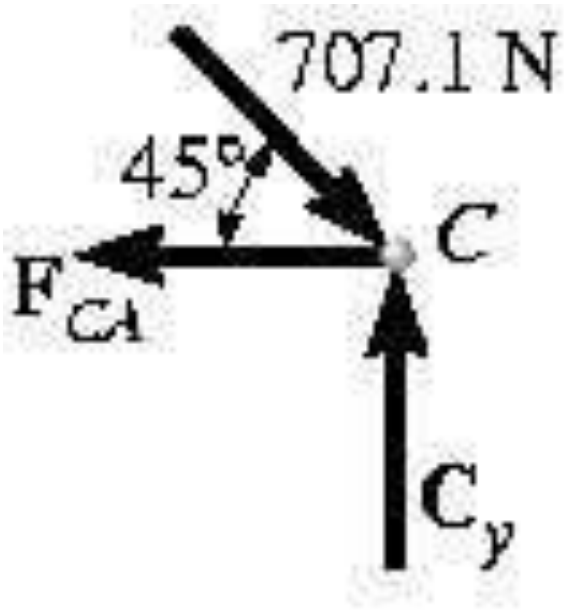
$$-F_{CA} + 707.1\cos 45^{\circ} N = 0$$

$$F_{CA} = 500N(T)$$

$$+\uparrow \sum F_y = 0;$$

$$C_y - 707.1\sin 45^{\circ} N = 0$$

$$C_y = 500N$$



(c)

6.2 The Method of Joints

Solution

Joint A

$$+\rightarrow \sum F_x = 0;$$

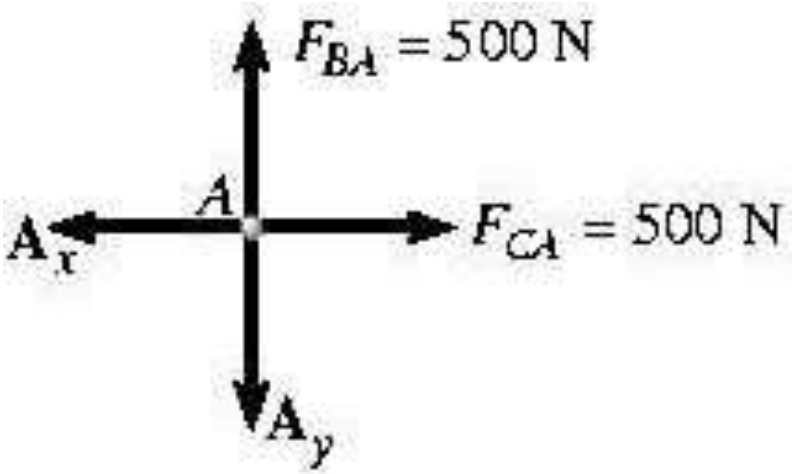
$$500N - A_x = 0$$

$$A_x = 500N$$

$$+\uparrow \sum F_y = 0;$$

$$500N - A_y = 0$$

$$A_y = 500N$$

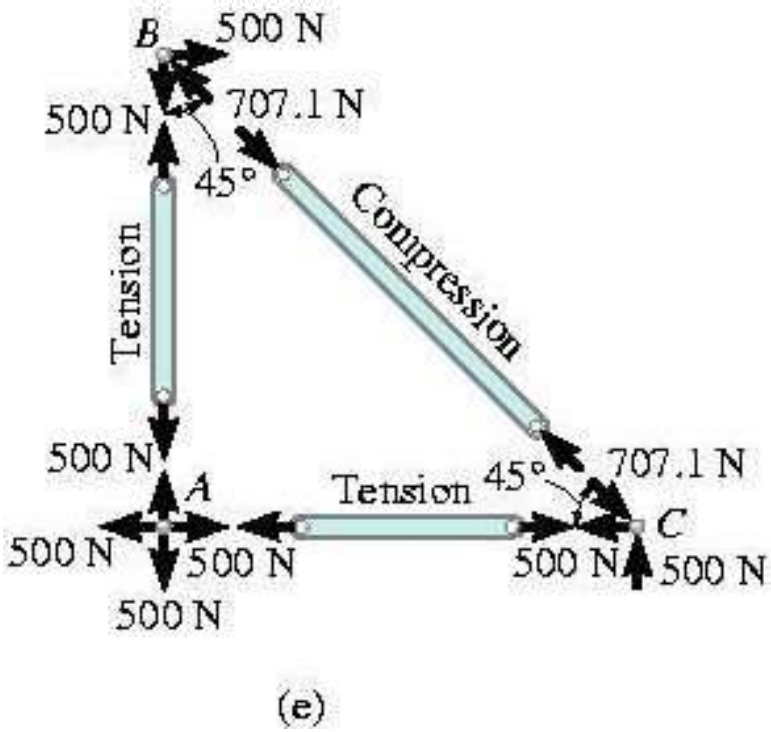


(d)

6.2 The Method of Joints

Solution

- FBD of each pin shows the effect of all the connected members and external forces applied to the pin
- FBD of each member shows only the effect of the end pins on the member

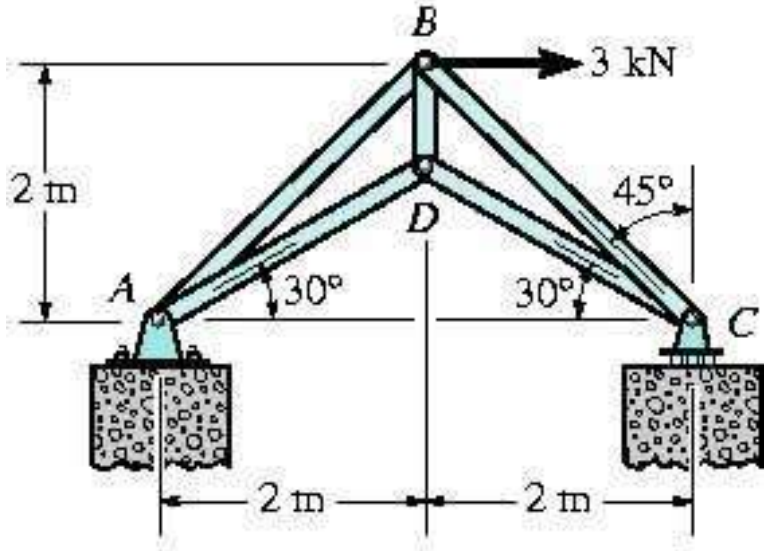


(e)

6.2 The Method of Joints

Example 6.2

Determine the forces acting in all the members of the truss.

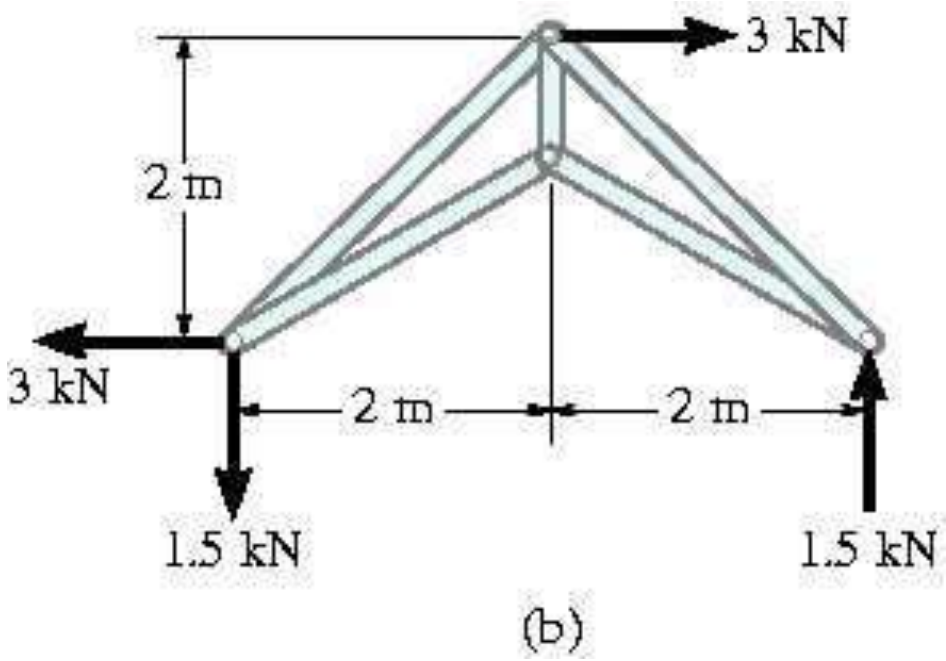


(a)

6.2 The Method of Joints

Solution

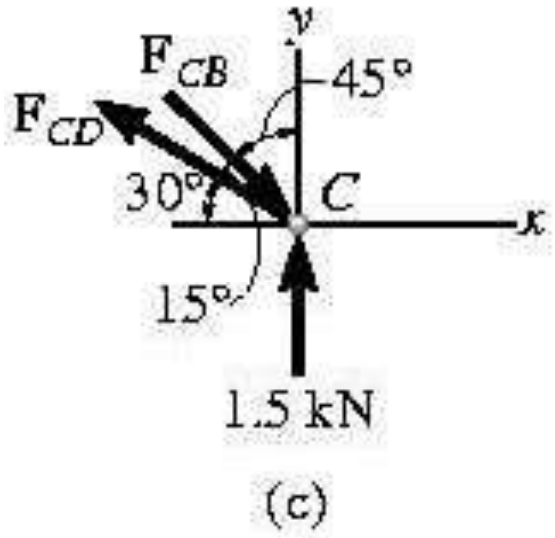
- Two unknowns at each joint
- Support reactions on the truss must be determined



6.2 The Method of Joints

Solution

Joint C



(c)

$$+\rightarrow \sum F_x = 0;$$

$$-F_{CD}\cos 30°kN + F_{CB}\sin 45°kN = 0$$

$$+\uparrow \sum F_y = 0;$$

$$1.5kN + F_{CD}\sin 30°kN - F_{CB}\cos 45°kN = 0$$

6.2 The Method of Joints

Solution

Joint C

$$\sum F_{x'} = 0;$$

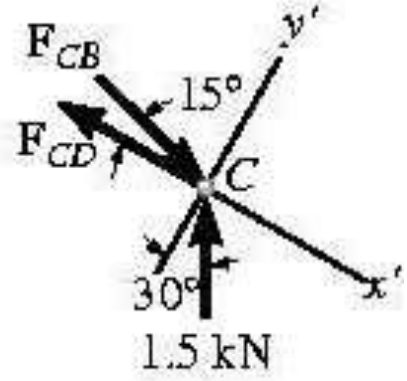
$$1.5\cos 30°kN - F_{CB}\sin 15°kN = 0$$

$$F_{CB} = 5.02kN$$

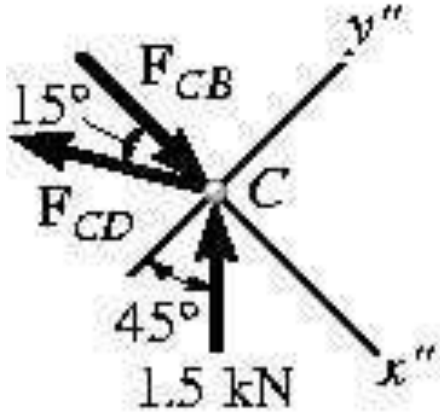
$$\sum F_{y''} = 0;$$

$$1.5\cos 30°kN - F_{CD}\sin 15°kN = 0$$

$$F_{CD} = 4.10kN$$



(d)



(e)

6.2 The Method of Joints

Solution

Joint D

$$+\rightarrow \sum F_x = 0;$$

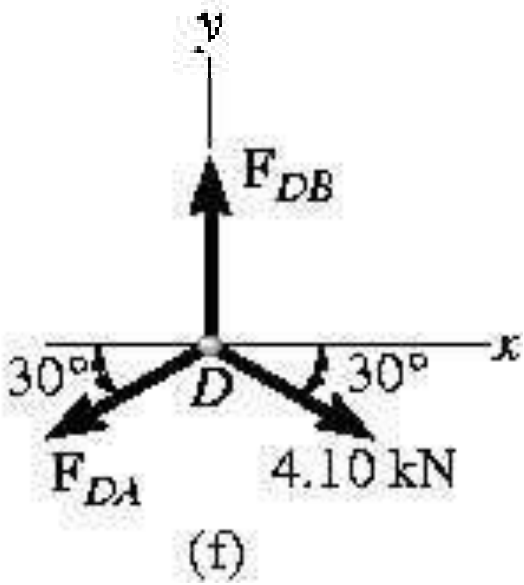
$$-F_{DA}\cos 30°kN + 4.10\cos 30°kN = 0$$

$$F_{DA} = 4.10kN\,(T)$$

$$+\uparrow \sum F_y = 0;$$

$$F_{DB} - 2(4.10\sin 30°kN) = 0$$

$$F_{DB} = 4.10kN\,(T)$$



(f)

6.2 The Method of Joints

Solution

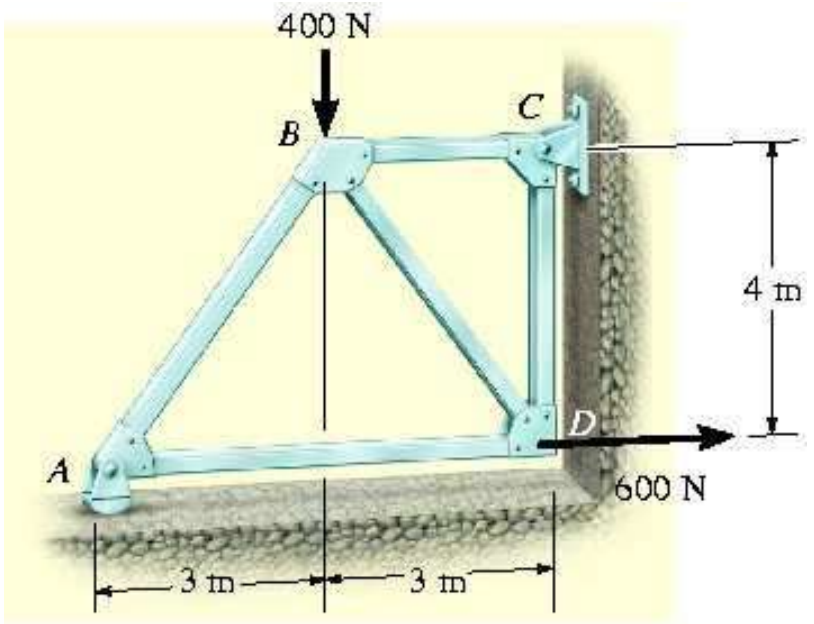
- From the FBD of joint B, sum the forces in the horizontal direction

$$F_{BA} = 0.776\text{kN (C)}$$

6.2 The Method of Joints

Example 6.3

Determine the force in each member of the truss. Indicate whether the members are in tension or compression.



(a)

6.2 The Method of Joints

Solution

Support Reactions

$$+\rightarrow \sum F_x = 0; 600N - C_x = 0$$

$$C_x = 600N$$

$$\sum M_C = 0; -A_y(6m) + 400N(3m) + 600N(4m) = 0$$

$$A_y = 600N$$

$$+\uparrow \sum F_y = 0; F_{DB} - 2(4.10\sin 30^{o}) = 0$$

$$F_{DB} = 4.10kN(T)$$

6.2 The Method of Joints

Solution

Joint A

$$+\rightarrow \sum F_x = 0;$$

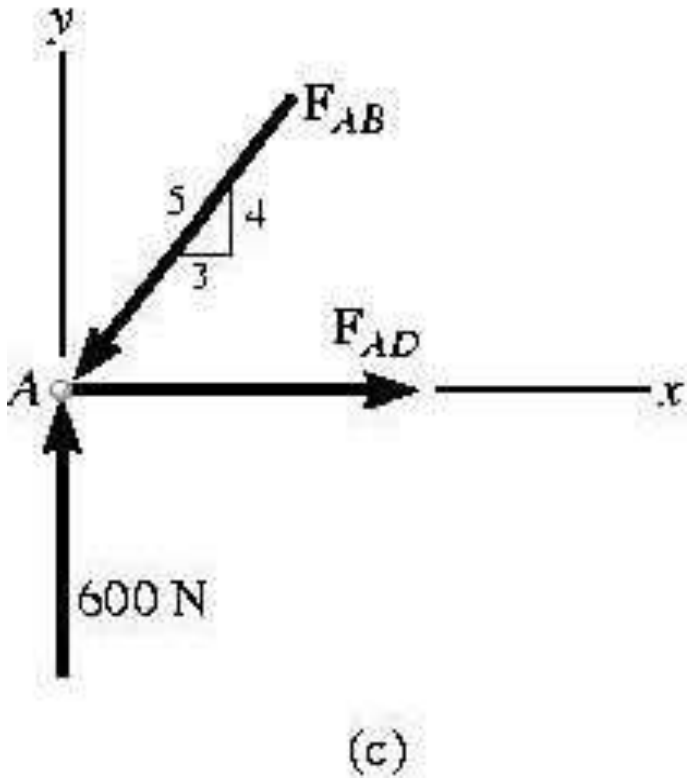
$$F_{AD} - \frac{3}{5}(750N) = 0$$

$$F_{AD} = 450N(T)$$

$$+\uparrow \sum F_y = 0;$$

$$600N - \frac{4}{5}F_{AB} = 0$$

$$F_{AB} = 750N(C)$$



(c)

6.2 The Method of Joints

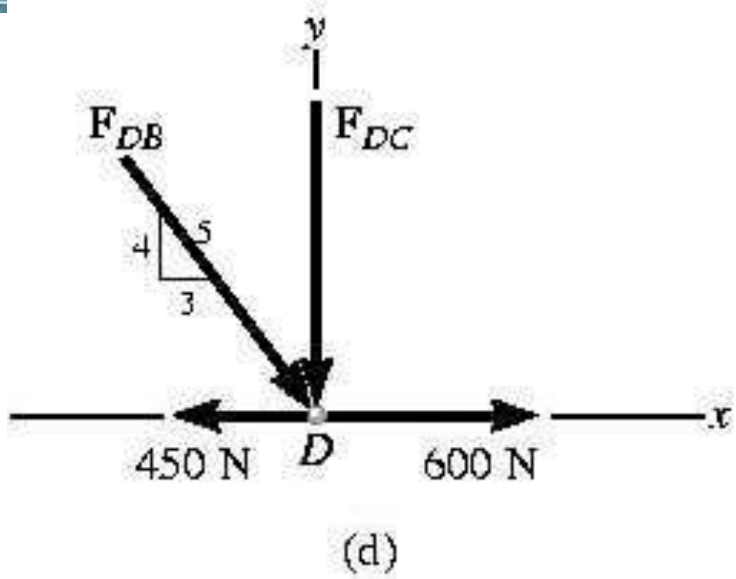
Solution

Joint D

$$+\rightarrow \sum F_x = 0;$$

$$-450N + \frac{3}{5}F_{DB} + 600N = 0$$

$$F_{DB} = -250N$$



(d)

Negative sign: reverse sense of $\mathbf{F}_{DB}$

$$F_{DB} = 250N(T)$$

$$+\uparrow \sum F_y = 0;$$

$$-F_{DC} - \frac{4}{5}(-250N) = 0$$

$$F_{DC} = 200N(C)$$

6.2 The Method of Joints

Solution

Joint C

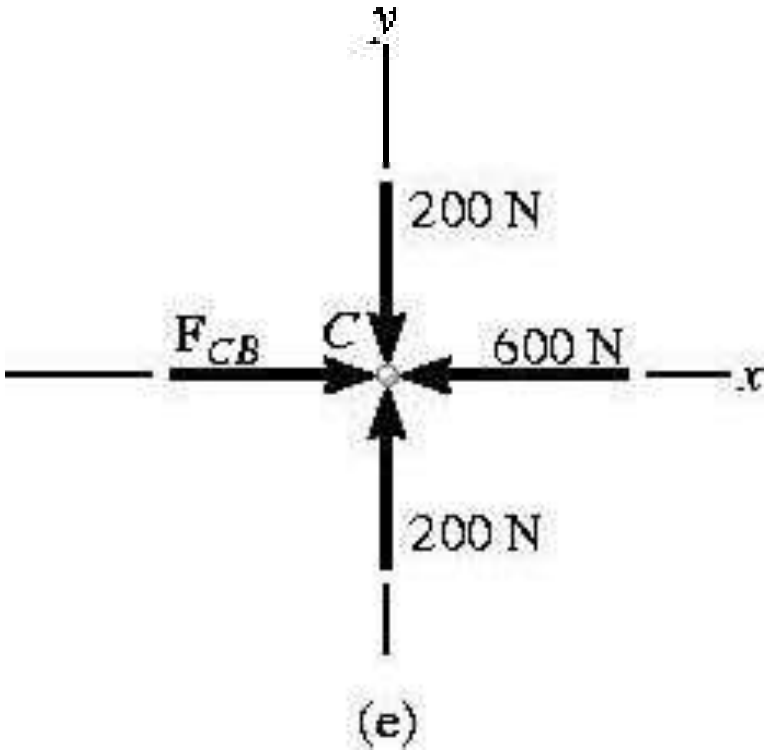
$$+\rightarrow \sum F_x = 0;$$

$$F_{CB} - 600N = 0$$

$$F_{CB} = 600N(C)$$

$$+\uparrow \sum F_y = 0;$$

$$200N - 200N \equiv 0(check)$$

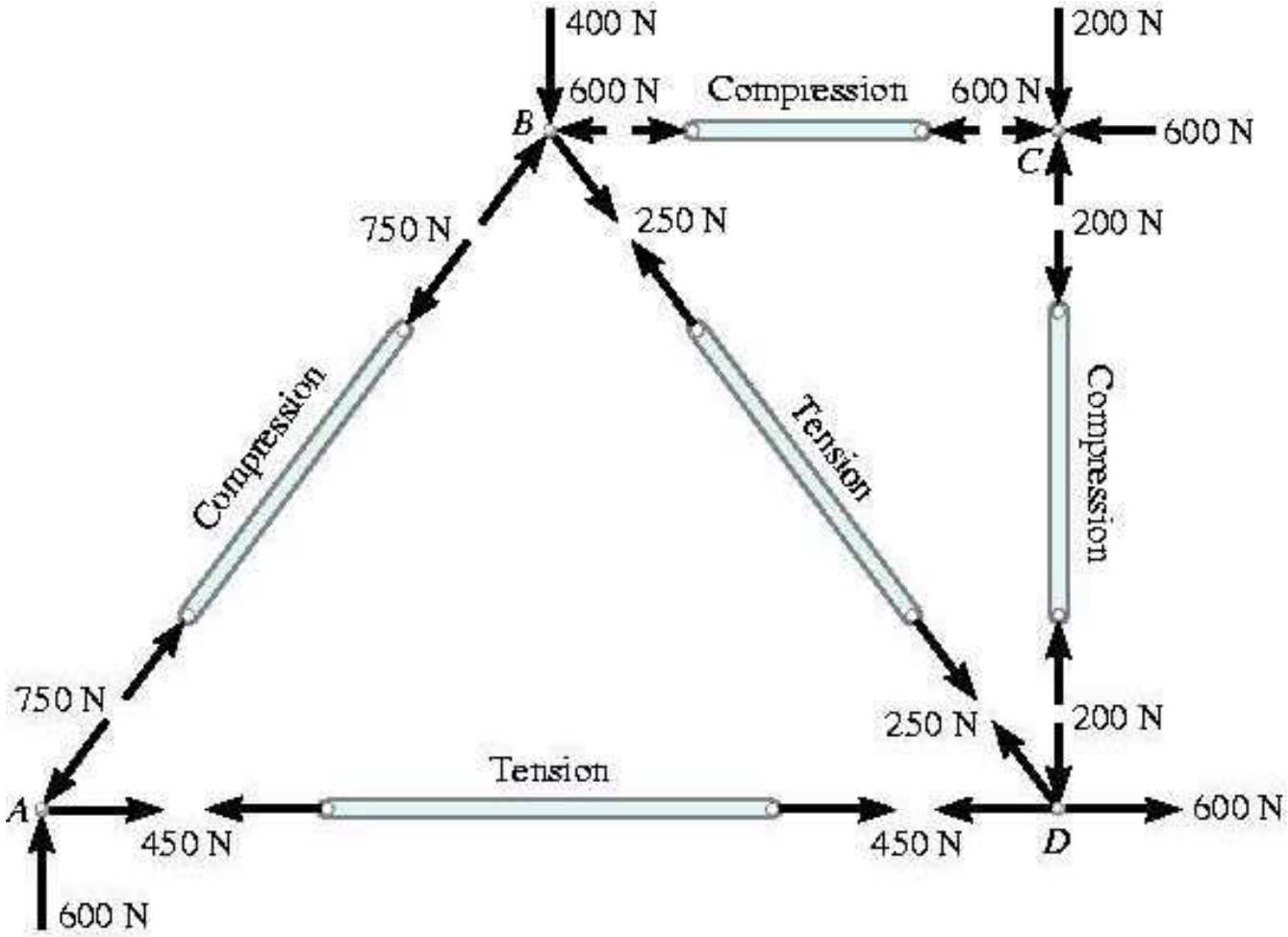


(e)

6.2 The Method of Joints

Solution

FBD



(f)